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# FRACTIONAL CALCULUS OPERATORS ASSOCIATED WITH THE PRODUCT OF $(p, q)$-EXTENDED BESSEL FUNCTION AND GENERALIZED K-STRUVE FUNCTION

**Riyaz Khan and Hemlata Saxena**

Department of Mathematics,
Career Point University,
Kota - 325003, Rajasthan, INDIA

E-mail : khan.riyazz84@gmail.com, saxenadrhemlata@gmail.com

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**Abstract:** Motivated by a recent work on Marichev-Saigo-Maeda fractional calculus operators associated with the generalized k-Struve function(Seema Kabra et al. [19] in Applied Mathematics and Nonlinear Sciences, 5(2),593-602), this paper establishes four theorem by using Marichev -Maeda-Saigo fractional integral and derivative operators involving the product of the $(p, q)$-Extended Bessel function and Generalized k-Struve function, supported by serveral auxillary lemmas. The results are expressed in terms of the ${}_{r+k}F_{s+k}$ and generalized k-Wright function ${}_{r}\psi_{s}{}^{k}$. Some new and known results are also obtained in special cases of main results.

**Keywords and Phrases:** MSM Fractional Calculus Operator, $(p, q)$-Extended Bessel function and Generalized k-Struve function, $(p, q)$-Extended generalized hypergeometric function and Generalized k-Wright function.

**2020 Mathematics Subject Classification:** Primary 26A33, Secondary 33C45, 33E12, 44A20.

## 1. Introduction

Fractional calculus is a branch of mathematics deals with various differential and integral operators. Fractional calculus operators are studied extensively due to their importance in applied problems of science and engineering. For our present study, we recall the generalized hypergeometric fractional integrals, introduced by Marichev [26], including the Saigo operators ([36], [37], [38]), and which were later on extended by Saigo and Maeda [39]. The Marichev-Saigo-Meada fractional calculus operator involving the product of some special functions defined thus.

**Maricchev-Saigo-Maeda Fractional Calculus Operators:** Saigo [36] defined the fractional integral and differential operators with the Gauss hyergeometric function as kernel, which are remarkable generalizations of the Riemann-Liouville (R-L) and Erdélyi-Kober fractional calculus operators [23]. Marichev [26] was introduced and studied fractional calculus operators which are the generalization of the Saigo operators, later generalized by Saigo and Maeda [39].
The some important definitions are defined by Singh, G. et. al [41] in the following manner as :

**Definition 1.** *Let $\zeta, \zeta', \delta, \delta', \tau \in \mathbb{C}$ and $x > 0$, then, for $\Re(\delta) > 0$,*

$$\left(I_{0,x}^{\zeta,\zeta',\delta,\delta',\tau} f\right)(x) = \frac{x^{-\zeta}}{\Gamma(\tau)} \int_0^x (x-t)^{\tau-1} t^{-\zeta'} F_3\left(\zeta, \zeta', \delta, \delta'; \tau; 1-\frac{t}{x}, 1-\frac{x}{t}\right) f(t)dt \tag{1.1}$$

*and*

$$\left(I_{x.\infty}^{\zeta,\zeta',\delta,\delta',\tau} f\right)(x) = \frac{x^{-\zeta}}{\Gamma(\tau)} \int_x^\infty (t-x)^{\tau-1} t^{-\zeta'} F_3\left(\zeta, \zeta', \delta, \delta'; \tau; 1-\frac{x}{t}, 1-\frac{t}{x}\right) f(t)dt \tag{1.2}$$

*where the function $f(t)$ is so constrained that the integrals in* (1.1)*and* (1.2) *exist.*
Let the M.S.M. type fractional integral operator be denoted by $I_{a,x}^{\zeta,\zeta',\delta,\delta',\tau} f(x)$, where subscripts $(a, x)$ indicate the integration limits and superscripts carry the operator parameters.
The above fractional integral operator in Eq.(1.1) and (1.2) can be written as follows:

$$\left(I_{x.\infty}^{\zeta,\zeta',\delta,\delta',\tau} f\right)(x) = \left(\frac{d}{dx}\right)^k \left(I_{x.\infty}^{\zeta,\zeta',\delta+k,\delta',\tau+k} f\right)(x) \tag{1.3}$$

and

$$\left(I_{x,\infty}^{\zeta,\zeta',\delta,\delta',\tau} f\right)(x) = \left(-\frac{d}{dx}\right)^k \left(I_{x,\infty}^{\zeta,\zeta',\delta,\delta'+k,\tau+k} f\right)(x) \tag{1.4}$$

where

$$(\Re(\tau) \leq 0; k = [-\Re(\tau) + 1]) .$$

**Proposition 1.** *$F_3(.)$ denoted by the Appell's hypergeometric function in two* *variables* [22] *defined as:*

$$F_3\left(\zeta,\zeta^{'},\delta,\delta^{'};\tau;u,v\right)=\sum_{m,n=0}^{\infty}\frac{(\zeta)_m(\zeta^{'})_n(\delta)_m(\delta^{'})_n}{(\delta)_{m+n}}\frac{u^m}{m!}\frac{v^n}{n!},(max\left\{|u|,|v|\right\}<1). \tag{1.5}$$

**Remark 1.** *The Appell function defined in Eq. (1.5) reduces to the Gauss hypergeometric function ${}_2F_1$ as given in following relations:*

$$F_3\left(\zeta,\tau,-\zeta^{'},\delta,\tau-\delta;\tau;u,v\right)={}_2F_1\left(\zeta,\delta;-\tau;u+v,-uv\right) \tag{1.6}$$

*also we have*

$$F_3\left(\zeta,0,\delta,\delta^{'},\tau;u,v\right)={}_2F_1\left(\zeta,\delta;\tau;u\right) \tag{1.7}$$

*and*

$$F_3\left(0,\zeta^{'},\delta,\delta^{'},\tau;u,v\right)={}_2F_1\left(\zeta^{'},\delta^{'};\tau;v\right) \tag{1.8}$$

**Definition 2.** *Let $\zeta,\zeta^{'},\delta,\delta^{'},\tau\in\mathbb{C}$ and $x>0$, then*

$$\begin{aligned}\left(D_{0,x}^{\zeta,\zeta^{'},\delta,\delta^{'},\tau}f\right)(x)&=\left(I_{0,x}^{-\zeta^{'}},-\zeta,-\delta^{'},-\delta,-\tau f\right)(x)\\&=\left(\frac{d}{dx}\right)^k\left(I_{0,x}^{-\zeta^{'}},-\zeta,-\delta^{'},-\delta+k,-\tau+kf\right)(x);(\Re(\tau)>0;k=[\Re(\tau)+1])\\&=\frac{1}{\Gamma(k-\tau)}\left(\frac{d}{dx}\right)^k x^{\zeta}\int_0^x(x-t)^{k-\tau-1}t^{\zeta}\\&\qquad\times F_3\left(-\zeta^{'},-\zeta,-\delta^{'},k-\delta;k-\tau;1-\frac{t}{x},1-\frac{x}{t}\right)f(t)dt\end{aligned} \tag{1.9}$$

*and*

$$\begin{aligned}\left(D_{x,\infty}^{\zeta,\zeta^{'},\delta,\delta^{'},\tau}f\right)(x)&=\left(I_{x,\infty}^{-\zeta^{'}},-\zeta,-\delta^{'},-\delta,-\tau f\right)(x)\\&=\left(-\frac{d}{dx}\right)^k\left(I_{x,\infty}^{-\zeta^{'}},-\zeta,-\delta^{'},-\delta+k,-\tau+kf\right)(x);(\Re(\tau)>0;k=[\Re(\tau)+1])\\&=\frac{1}{\Gamma(k-\tau)}\left(-\frac{d}{dx}\right)^k(x)^{\zeta}\int_0^x(t-x)^{k-\tau-1}t^{\zeta^{'}}\\&\qquad\times F_3\left(-\zeta^{'},-\zeta,-\delta^{'},k-\delta;k-\tau;1-\frac{x}{t},1-\frac{t}{x}\right)f(t)dt\end{aligned} \tag{1.10}$$

In view of the above reduction formula as given Eq.(1.7),the The general fractional calculus operators reduces to the Saigo operator defined as follows [36].

**Definition 3.** *For $x > 0, \zeta, \delta, \delta', \tau \in \mathbb{C}$ and $\Re(\zeta) > 0$, then*

$$\left(I_{0,x}^{\zeta,\delta,\tau} f\right)(x) = \frac{x^{-\zeta-\delta}}{\Gamma(\zeta)} \int_0^x (x-t)^{\zeta-1} {}_2F_1\left(\zeta+\delta, -\tau; \zeta; 1-\frac{t}{x}\right) f(t)dt \tag{1.11}$$

*and*

$$\left(I_{x,\infty}^{\zeta,\delta,\tau} f\right)(x) = \frac{1}{\Gamma(\zeta)} \int_x^\infty (t-x)^{\zeta-1} t^{-\zeta-\delta} {}_2F_1\left(\zeta+\delta, -\tau; \zeta; 1-\frac{x}{t}\right) f(t)dt \tag{1.12}$$

**Proposition 2.** *${}_2F_1(\zeta, \delta; \tau; z)$ denoted by the Gauss hypergeometric function* [23] *defined for $u \in \mathbb{C}, |u| < 1$.*

$${}_2F_1(\zeta, \delta; \tau; u) = \sum_{n=0}^{\infty} \frac{(\zeta)_n (\delta)_n}{(\tau)_n} \frac{u^n}{n!} \tag{1.13}$$

**Definition 4.** *Let $\zeta, \delta, \tau \in \mathbb{C}$ and $x > 0$, then*

$$\left(D_{0,x}^{\zeta,\delta,\tau} f\right)(x) = \left(I_{0,x}^{-\zeta,-\delta,\zeta+\tau} f\right)(x) = \left(\frac{d}{dx}\right)^k \left(I_{0,x}^{-\zeta+k,-\delta-k,\zeta+\tau-k} f\right)(x) \tag{1.14}$$

*and*

$$\left(D_{x,\infty}^{\zeta,\delta,\tau} f\right)(x) = \left(I_{x,\infty}^{-\zeta,-\delta,\zeta+\tau} f\right)(x) = \left(-\frac{d}{dx}\right)^k \left(I_{x,\infty}^{-\zeta+k,-delta-k,\zeta+\tau} f\right)(x) \tag{1.15}$$

*where*

$$(\Re(\zeta) > 0; k = [\Re(\zeta) + 1])$$

*and $\Re(\zeta)$is the integer part of $\Re(\zeta)$.*
If we take $\delta = 0$ in Eqs.(1.11), (1.12), (1.14) and (1.15) we get the so-called Erdely-Kober fractional integral and derivative give definite of Erdely-Kober operators defined as follows ([24], [27]).

**Generalized k-Struve function**

The generalized k-Struve function defined by Nisar et al. [29] in the following manner:

$$S_{v,c}^k(t) = \sum_{n=0}^{\infty} \frac{(-c)^n}{\Gamma_k\left(nk + v + \frac{3k}{2}\right)\Gamma\left(n + \frac{3}{2}\right)} \left(\frac{t}{2}\right)^{2n+\frac{v}{k}+1} \left(k \in \Re^+; c \in \Re; v > -1\right) \tag{1.16}$$

where the generalized k-Struve function $S^k_{v,c}(t)$ convergence for all complex values of $t$ for details by Nisar et.al [30].

**Proposition 3.** *The k-Gamma function defined by Daiz and Pariguan* [11] *in the following manner:*

$$\Gamma_k(z) = \int_0^\infty t^{z-1} e^{-\frac{t^k}{k}} dt, z \in \mathbb{C}, k > 0 \tag{1.17}$$

**Proposition 4.** *If* $z \in \mathbb{C}$ *and* $k \in \Re$,*then the following identity is true*

$$\Gamma_k(z+k) = z\Gamma_k(z) \tag{1.18}$$

*and*

$$\Gamma_k(z) = k^{\frac{z}{k}-1}\Gamma\left(\frac{z}{k}\right) \tag{1.19}$$

**Remark 2.** *If we taking* $k \to 1$ *and* $c = 1$ *in* (1.16) *reduces to well-known Struve function of order* $v$ *defined by* [5] *as*

$$H_v(z) = \sum_{r=0}^{\infty} \frac{(-1)^r}{\Gamma\left(r+v+\frac{3}{2}\right)\Gamma\left(r+\frac{3}{2}\right)} \left(\frac{z}{2}\right)^{2r+v+1} \tag{1.20}$$

For further details about Struve functions,their generalizations and its properties, the esteemed reader is invited to consider references ([6, 17, 21, 31, 32, 42, 46, 51]). Also Diaz et al. ([12], [13]) introduced the k-gamma function, k-beta function and Pochhammer k-symbols.

$(p, q)$ **- Extended Bessel function**

The $(p, q)$-extended Bessel function $J_{\nu,p,q}(z)$ of the first kind of order $\nu$ is defined as follows [18].

$$J_{\nu,p,q}(z) = \frac{\sqrt{\pi}}{\Gamma(\nu+1)} \sum_{m=0}^{\infty} \frac{(-1)^m B(m+\frac{1}{2}, \nu+\frac{1}{2}; p, q)}{m!\Gamma(m+\frac{1}{2})B(\frac{1}{2}, \nu+\frac{1}{2})} \left(\frac{z}{2}\right)^{2m+\nu} \tag{1.21}$$

where $min\left\{\Re(p), \Re(q)\right\} \geq 0$ and $\Re(\nu) > -1$ when $p = q = 0$.

**Proposition 5.** *The* $(p, q)$*-extended Beta function defined by Choi et al.[10] in the following manner as:*

$$B\left(u, v; p, q\right) = \int_0^1 t^{u-1}(1-t)^{v-1} e^{\left(-\frac{p}{t}-\frac{q}{1-t}\right)} \tag{1.22}$$

$$(min\{\Re(u), \Re(v)\} > 0; min\left\{\Re(p), \Re(q)\right\} \geq 0)$$

It should be remarked here that the existing literature on the subject contains much more general extensions of the classical Beta function,especially in the case when $p = q$ ([25], [43]).
For $p = q = 1$, the $(p, q)$-extended Bessel function of the first kind $J_{\nu,p,q}(z)$ and the $(p, q)$-extended Beta function $B(x, y; p.q)$ reduces Bessel function $J_\nu(z)$ of the first kind and the classical Beta function $B(x, y)$ , respectively.

**$(p, q)$ - extended Gauss hypergeometric function**

The $(p, q)$-extended Gauss hypergeometric function $F_{p,q}$ is defined as follows [10]:

$$F_{p,q}(u, v; w; z) = \sum_{n=0}^{\infty}(u)_n \frac{B(v + n, w - v; p, q)}{B(v, w - v)} \frac{z^n}{n!} \tag{1.23}$$

where $|z| < 1$ and $\Re(w) > \Re(v) > 0$.

**$(p, q)$ -extended Generalized hypergeometric function**

The $(p, q)$-extended generalized hypergeometric functions. We defined in the series from as

$${}_{r+k}F_{s+k}\left[\begin{array}{c} a_1, ..., a_r, A_i, ..., A_k; \\ c_i, ...c_s, C_1, ..., C_k; \end{array} z; p, q\right] = \sum_{n\geq 0}^{\infty} \frac{\prod_{j=1}^{r}(a_j)_n}{\prod_{j=1}^{s}(c_j)_n} \prod_{j=1}^{k} \frac{B(A_j + n, C_j - A_j; p, q)}{B(A_j, C_j - A_j)} \frac{z^n}{n!} \tag{1.24}$$

The special case of the defining series (1.24) when e.g. k = r = s+1 = 1; and *a fortiori* $a_1 = a, A_1 = b, C_1 = c$ becomes the already known $(p, q)$-extended Gausian hypergeometric function ([1, 7, 8, 9, 10, 34, 35]).

**Fox-Wright function**

An interesting generalized hypergeometric function of one variable [44] and further generalization of the series ${}_rF_s$ were given by Fox [15] and Wright ([48-50]):

$$\begin{aligned} {}_r\psi_s(x) &= {}_r\psi_s\left[\begin{array}{c} (a_i, C_i)_{1,r}; \\ (b_i, D_i)_{1,s}; \end{array} x\right] \\ &= {}_r\psi_s\left[\begin{array}{c} (a_1, C_1), (a_2, C_2), ..., (a_r, C_r); \\ (b_1, D_1), (b_2, D_2), ..., (b_s, D_s); \end{array} x\right] \\ &= \sum_{n=0}^{\infty} \frac{\prod_{i=1}^{r} \Gamma(a_i + nC_i)}{\prod_{j=1}^{s} \Gamma(b_j + nD_j)} \frac{x^n}{n!}, x \in \mathbb{C} \\ &= \sum_{n=0}^{\infty} \frac{\Gamma(a_1 + nC_1)\Gamma(a_2 + nC_2)...\Gamma(a_p + nC_p)}{\Gamma(b_1 + nD_1)\Gamma(b_2 + nD_2)...\Gamma(b_q + nD_q)} \frac{x^n}{n!}, x \in \mathbb{C} \end{aligned} \tag{1.25}$$

The generalized from of the above Wright function (1.25) was given by Gehlot and Prajapati [16], named as generalized K-Wright function which is defined as

$$\begin{aligned}
{}_r\psi_s^k(x) &= {}_r\psi_s^k\left[\begin{array}{c}(a_i,C_i)_{1,r};\\ (b_i,D_i)_{1,s};\end{array} x\right] \\
&= {}_r\psi_s^k\left[\begin{array}{c}(a_1,C_1),(a_2,C_2),...,(a_r,C_r);\\ (b_1,D_1),(b_2,D_2),...,(b_s,D_s);\end{array} x\right] \\
&= \sum_{n=0}^{\infty}\frac{\prod_{i=1}^{r}\Gamma_k(a_i+nC_i)}{\prod_{j=1}^{s}\Gamma_k(b_j+nD_j)}\frac{x^n}{n!}, x\in\mathbb{C} \\
&= \sum_{n=0}^{\infty}\frac{\Gamma_k(a_1+nC_1)\Gamma_k(a_2+nC_2)...\Gamma_k(a_r+nC_r)}{\Gamma_k(b_1+nD_1)\Gamma_k(b_2+nD_2)...\Gamma_k(b_x+nD_s)}\frac{x^n}{n!}, x\in\mathbb{C}
\end{aligned} \tag{1.26}$$

**Extended Wright Generalized hypergeometric function**

The extended Wright generalized hypergeometric function defined by Sharma and Deci [40] in the following manner as:

$${}_{r+1}\psi_{s+1}\left[\begin{array}{c}(m_i,M_i)_{1,r},(\eta+1);\\ (n_i,N_i)_{1,s},(\tau,1);\end{array} x;p\right] = \frac{1}{\Gamma(\tau-\eta)}\sum_{k=0}^{\infty}\frac{\prod_{i=1}^{r}\Gamma(m_i+kM_i)B_p(\eta+k,\tau-\eta)}{\prod_{j=1}^{s}\Gamma(n_j+kN_j)}\frac{x^k}{k!} \tag{1.27}$$

$$\left(\Re(\tau)>\Re(\eta)>0, \Re(p)>0; r,s\in\mathbb{C}; M_i,N_j\in\Re^+; i=1,...,r, j=1,..s\right)$$

**Some Required Lemma**

The required lemmas defined by Saigo, M. [36] in the following manner as:

**Lemma 1.** *Let $\zeta,\zeta',\delta,\delta',\tau,\lambda\in\mathbb{C}, x>0$ be such that $\Re(\tau)>0$; then the following formulas hold true:*

$$\left(I_{0,x}^{\zeta,\zeta',\delta,\delta',\tau}t^{\lambda-1}\right)(x) = \frac{\Gamma(\lambda)\Gamma\left(\lambda+\tau-\zeta-\zeta'-\delta\right)\Gamma\left(\lambda+\delta'-\zeta'\right)}{\Gamma(\lambda+\delta')\Gamma(\lambda+\tau-\zeta-\zeta')\Gamma(\lambda+\tau-\zeta'-\delta)}x^{\lambda+\tau-\zeta-\zeta'-1}$$
$$\left(\Re(\lambda)>max\{0,\Re\left(\zeta+\zeta'+\delta-\tau\right),\Re\left(\zeta'-\delta'\right)\}\right). \tag{1.28}$$

*and*

$$\left(I_{x,\infty}^{\zeta,\zeta',\delta,\delta',\tau}t^{\lambda-1}\right)(x) = \frac{\Gamma(1-\lambda-\delta)\Gamma\left(1-\lambda-\tau+\zeta+\zeta'\right)\Gamma\left(1-\lambda-\tau+\zeta+\delta'\right)}{\Gamma(1-\lambda)\Gamma(1-\lambda-\tau+\zeta+\zeta'+\delta')\Gamma(1-\lambda+\zeta-\delta)} \times x^{\lambda+\tau-\zeta-\zeta'-1} \tag{1.29}$$

$$\left(\Re(\lambda)<1+min\{\Re(-\delta),\Re\left(\zeta+\zeta'-\tau\right),\Re\left(\zeta+\delta'-\tau\right)\}\right)$$

**Lemma 2.** *Let $\zeta, \zeta^{'}, \delta, \delta^{'}, \tau, \lambda \in \mathbb{C}, x > 0$ be such that $\Re(\tau) > 0$; then the following formulas hold true:*

$$\left(D_{0,x}^{\zeta,\zeta^{'},\delta,\delta^{'},\tau} t^{\lambda-1}\right)(x) = \frac{\Gamma(\lambda)\Gamma\left(\lambda-\tau+\zeta+\zeta'+\delta'\right)\Gamma(\lambda-\delta+\zeta)}{\Gamma(\lambda-\delta)\Gamma(\lambda-\tau+\zeta+\zeta')\Gamma(\lambda-\tau+\zeta+\delta')} \times x^{\lambda-\tau+\zeta+\zeta^{'}-1} \tag{1.30}$$

$$\left(\Re(\lambda) > max\{0, \Re\left(\tau-\zeta-\zeta^{'}-\delta^{'}\right), \Re(\delta-\zeta)\}\right).$$

*and*

$$\left(D_{x,\infty}^{\zeta,\zeta^{'},\delta,\delta^{'},\tau} t^{\lambda-1}\right)(x) = \frac{\Gamma\left(1-\lambda+\delta'\right)\Gamma\left(1-\lambda+\tau-\zeta-\zeta'\right)\Gamma\left(1-\lambda+\tau-\zeta'-\delta\right)}{\Gamma(1-\lambda)\Gamma(1-\lambda+\tau-\zeta-\zeta'-\delta)\Gamma(1-\lambda-\zeta'+\delta')} \times x^{\lambda-\tau+\zeta+\zeta^{'}-1} \tag{1.31}$$

$$\left(\Re(\lambda) < 1 + min\{\Re\left(\delta^{'}\right), \Re\left(\tau-\zeta+\zeta^{'}\right), \Re\left(\tau-\zeta^{'}-\delta\right)\}\right)$$

## 2. Main Results

In this section, the following for four theorem involving the product of $(p, q)$-Extended Bessel function and Generalized k-Struve function using by Marichev-Saigo- Maeda fractional integral and derivative operators are established here as main results.

**Theorem 1.** *Let $\zeta, \zeta^{'}, \delta, \delta^{'}, \tau, \frac{\eta}{k}, \mu \in \mathbb{C}$ and $k > 0$ be such that $\Re(\tau) > 0, \Re(\mu) > 0, \Re(\frac{\eta}{k}) > max\{0, \Re(\zeta^{'}-\delta^{'}), \Re(\zeta+\zeta^{'}+\delta-\tau)\}$.Also let $c \in \Re; v > -1$,then for $t > 0$.*

$$\left(I_{0,x}^{\zeta,\zeta^{'},\delta,\delta^{'},\tau}\left[t^{\frac{\eta}{k}-1} S_{v,c}^{k}(t) J_{\nu,p,q}(t^{\mu})\right]\right)(x) = x^{\tau-\zeta-\zeta^{'}+\frac{\eta}{k}+\frac{v}{k}+\mu\nu} \frac{1}{\Gamma(\nu+1)} k^{\tau+\frac{1}{2}} \left(\frac{1}{2}\right)^{\nu+\frac{v}{k}+1}$$

$${}_1F_2\left[\begin{matrix}\frac{1}{2}\\ \frac{1}{2}, \nu+1\end{matrix}\middle| -\frac{x^{2\mu}}{4}; p, q\right] \times$$

$${}_4\psi_5^k\left[\begin{matrix}(v+\eta+k+2\mu mk+\mu\nu k, 2k),\\ (v+\eta+k+\tau k-\zeta k-\zeta^{'}k-\delta k+2\mu mk+\mu\nu k, 2k),\\ (v+\eta+k+2\mu mk+\mu\nu k+\delta^{'}k-\zeta^{'}k, 2k), (1,1) & ;\\ (v+\eta+k+2\mu mk+\mu\nu k+\delta^{'}k, 2k),\\ (v+\eta+k+\tau k-\zeta k-\zeta^{'}k+2\mu mk+\mu\nu k, 2k),\\ (v+\eta+k+2\mu mk+\mu\nu k+\tau k-\zeta^{'}k-\delta k, 2k),\\ (v+\frac{3k}{2}, k), (\frac{3k}{2}, k) & ;\end{matrix} \; -\frac{x^2ck}{4}\right] \tag{2.1}$$

**Proof.** We denotes the left-hand sides of theorem (1) by $\Delta_1$,

$$\Delta_1 = \left(I_{0,x}^{\zeta,\zeta',\delta,\delta',\tau}\left[t^{\frac{\eta}{k}-1}S_{v,c}^{k}(t)J_{\nu,p,q}(t^{\mu})\right]\right)(x) \tag{2.2}$$

Using formula (1.16) and (1.21) in the right -hand side of Eq.(2.2) then change the order of the Integration and summation,we find that

$$\begin{aligned}\Delta_1 =& \frac{\sqrt{\pi}}{\Gamma(\nu+1)}\sum_{m=0}^{\infty}\frac{(-1)^m B(m+\frac{1}{2},\nu+\frac{1}{2};p,q)}{m!\Gamma(m+\frac{1}{2}B(\nu+\frac{1}{2})}\left(\frac{1}{2}\right)^{2m+\nu}\\ &\sum_{n=0}^{\infty}\frac{(-c)^n}{\Gamma_k\left(nk+v+\frac{3k}{2}\right)\Gamma\left(n+\frac{3}{2}\right)}\left(\frac{1}{2}\right)^{2n+\frac{v}{k}+1}\\ &\left(I_{0,x}^{\zeta,\zeta',\delta,\delta',\tau}\left[t^{\frac{\eta}{k}-1+\frac{v}{k}+2m\mu+\mu\nu+2n+1-1}\right]\right)(x)\end{aligned} \tag{2.3}$$

On applying lemma (1.28) in Eq.(2.3),we get

$$\begin{aligned}\Delta_1 =& x^{\mu\nu+\frac{\eta}{k}+\frac{v}{k}+\tau-\zeta-\zeta'}\left(\frac{1}{2}\right)^{\nu+\frac{v}{k}+1}\frac{\sqrt{\pi}}{\Gamma(\nu+1)}\\ &\times\sum_{m=0}^{\infty}\frac{(-1)^m B(m+\frac{1}{2},\nu+\frac{1}{2};p,q)}{m!\Gamma(m+\frac{1}{2}B(\nu+\frac{1}{2})}\left(-\frac{x^{2\mu}}{4}\right)^{m}\\ &\times\sum_{n=0}^{\infty}\frac{(-c)^n}{\Gamma_k\left(nk+v+\frac{3k}{2}\right)\Gamma\left(n+\frac{3}{2}\right)}\left(-\frac{cx^2}{4}\right)^{n}\\ &\times\frac{\Gamma\left(\frac{\eta}{k}+\frac{v}{k}+2m\mu+\mu\nu+2n+1+\tau-\zeta-\zeta'-\delta\right)}{\Gamma\left(\frac{\eta}{k}+\frac{v}{k}+2m\mu+\mu\nu+2n+1+\tau-\zeta-\zeta'\right)}\\ &\times\frac{\Gamma\left(\frac{\eta}{k}+\frac{v}{k}+2m\mu+\mu\nu+2n+1+\delta'-\zeta'\right)}{\Gamma\left(\frac{\eta}{k}+\frac{v}{k}+2m\mu+\mu\nu+2n+1+\tau-\zeta'-\delta\right)}\\ &\times\frac{\Gamma\left(\frac{\eta}{k}+\frac{v}{k}+2m\mu+\mu\nu+2n+1+\delta'-\zeta'\right)}{\Gamma\left(\frac{\eta}{k}+\frac{v}{k}+2m\mu+\mu\nu+2n+1+\tau-\zeta'-\delta\right)}\end{aligned} \tag{2.4}$$

Now using Eq.(1.19) in (2.4) ,we get

$$\begin{aligned}\Delta_1 =& x^{\mu\nu+\frac{\eta}{k}+\frac{v}{k}+\tau-\zeta-\zeta'}k^{\tau-2\zeta+\frac{1}{2}}\left(\frac{1}{2}\right)^{\nu+\frac{v}{k}+1}\frac{1}{\Gamma(\nu+1)}\\ &\times\sum_{m=0}^{\infty}\frac{(-1)^m B(m+\frac{1}{2},\nu+\frac{1}{2};p,q)}{m!\Gamma(m+\frac{1}{2}B(\nu+\frac{1}{2})}\left(-\frac{x^{2\mu}}{4}\right)^{m}\\ &\times\sum_{n=0}^{\infty}\frac{(-c)^n}{\Gamma_k\left(nk+v+\frac{3k}{2}\right)\Gamma\left(n+\frac{3}{2}\right)}\left(-\frac{ckx^2}{4}\right)^{n}\end{aligned}$$

$$\times \frac{\Gamma_k\left(\eta+v+2m\mu k+\mu\nu k+2nk+k\right)}{\Gamma_k\left(\eta+v+2m\mu k+\mu\nu k+2nk+k+\delta' k\right)}$$
$$\times \frac{\Gamma_k\left(\eta+v+2m\mu k+\mu\nu k+2nk+k+\tau k-\zeta k-\zeta' k-\delta k\right)}{\Gamma_k\left(\eta+v+2m\mu k+\mu\nu k+2nk+k+\tau k-\zeta k-\zeta' k\right)} \quad (2.5)$$
$$\times \frac{\Gamma_k\left(\eta+v+2m\mu k+\mu\nu k+2nk+k+\delta' k-\zeta' k\right)}{\Gamma_k\left(\eta+v+2m\mu k+\mu\nu k+2nk+k+\tau k-\zeta' k-\delta k\right)}$$

Now using Eq.(1.24) in Eq.(2.5) ,then we get

$$\Delta_1 = x^{\mu\nu+\frac{\eta}{k}+\frac{v}{k}+\tau-\zeta-\zeta'} k^{\tau-2\zeta+\frac{1}{2}} \left(\frac{1}{2}\right)^{\nu+\frac{v}{k}+1} \frac{1}{\Gamma(\nu+1)} \times {}_1F_2\left[\begin{array}{c}\frac{1}{2}\\ \frac{1}{2},\nu+1\end{array}\middle| -\frac{x^{2\mu}}{4};p,q\right]$$
$$\times \sum_{n=0}^{\infty} \frac{(-c)^n}{\Gamma_k\left(nk+v+\frac{3k}{2}\right)\Gamma\left(n+\frac{3}{2}\right)} \left(-\frac{ckx^2}{4}\right)^n$$
$$\times \frac{\Gamma_k\left(\eta+v+2m\mu k+\mu\nu k+2nk+k\right)}{\Gamma_k\left(\eta+v+2m\mu k+\mu\nu k+2nk+k+\delta' k\right)} \quad (2.6)$$
$$\times \frac{\Gamma_k\left(\eta+v+2m\mu k+\mu\nu k+2nk+k+\tau k-\zeta k-\zeta' k-\delta k\right)}{\Gamma_k\left(\eta+v+2m\mu k+\mu\nu k+2nk+k+\tau k-\zeta k-\zeta' k\right)}$$
$$\times \frac{\Gamma_k\left(\eta+v+2m\mu k+\mu\nu k+2nk+k+\delta' k-\zeta' k\right)}{\Gamma_k\left(\eta+v+2m\mu k+\mu\nu k+2nk+k+\tau k-\zeta' k-\delta k\right)}$$

Using the definition of (1.26) in (2.6),we at once arrive at the desired result.

**Theorem 2.** *Let* $\zeta,\zeta',\delta,\delta',\tau,\frac{\eta}{k},\mu \in \mathbb{C}$ *and* $k>0, k\in\Re^+, |\frac{1}{t}|\le 1$ *be such that* $\Re(\tau)>0, \Re(\mu)>0, \Re(\frac{\eta}{k}) < 1+min\{\Re(-\delta), \Re\left(\zeta+\zeta'-\tau\right), \Re\left(\zeta+\delta'-\tau\right)\}$. *Also let* $c\in\Re; v>-1$,*then for* $t>0$

$$\left(I_{x,\infty}^{\zeta,\zeta',\delta,\delta',\tau}\left[t^{\frac{\eta}{k}-1} S_{v,c}^{k}(t) J_{\nu,p,q}(\frac{1}{t^\mu})\right]\right)(x) = \frac{x^{\tau-\zeta-\zeta'+\frac{\eta}{k}+\frac{v}{k}-\mu\nu}}{\Gamma(\nu+1)} k^{\tau+\frac{1}{2}} \left(\frac{1}{2}\right)^{\nu+\frac{v}{k}+1}$$
$${}_1F_2\left[\begin{array}{c}\frac{1}{2}\\ \frac{1}{2},\nu+1\end{array}\middle| -\frac{1}{4x^{2\mu}};p,q\right] \times$$

$$ {}_4\psi_5^k\left[\begin{array}{cc}\begin{array}{c}(2\mu mk+\mu\nu k-\eta-v-\delta k,-2k),\\ (2\mu mk+\mu\nu k-v-\eta-\tau k+\zeta k+\zeta' k,-2k),\\ (2\mu mk+\mu\nu k-\eta-v-\tau k+\zeta k+\delta' k,-2k),(1,1)\end{array} & ;\\ \begin{array}{c}(2\mu mk+\mu\nu k-v-\eta,-2k),\\ (2\mu mk+\mu\nu k-v-\eta-\tau k+\zeta k+\zeta' k+\delta' k,-2k),\\ (2\mu mk+\mu\nu k-v-\eta+\zeta k-\delta k,-2k)\\ ,(v+\frac{3k}{2},k),(\frac{3k}{2},k)\end{array} & ;\end{array} -\frac{x^2ck}{4}\right] \quad (2.7)$$

**Proof.** We denotes the left-hand sides of theorem (2) by $\Delta_2$,

$$\Delta_2 = \left( I_{x,\infty}^{\zeta,\zeta',\delta,\delta',\tau} \left[ t^{\frac{\eta}{k}-1} S_{v,c}^{k}(t) J_{\nu,p,q}(\frac{1}{t^{\mu}}) \right] \right)(x) \tag{2.8}$$

Using formula (1.16) and (1.21) in the right -hand side of Eq.(2.8) then change the order of the Integration and summation,we find that

$$\begin{aligned}
\Delta_2 = & \frac{\sqrt{\pi}}{\Gamma(\nu+1)} \sum_{m=0}^{\infty} \frac{(-1)^m B(m+\frac{1}{2}, \nu+\frac{1}{2}; p, q)}{m!\Gamma(m+\frac{1}{2}) B(\frac{1}{2}, \nu+\frac{1}{2})} \left(\frac{1}{2}\right)^{2m+\nu} \\
& \times \sum_{n=0}^{\infty} \frac{(-c)^n}{\Gamma_k\left(nk+v+\frac{3k}{2}\right)\Gamma\left(n+\frac{3}{2}\right)} \\
& \times \left(\frac{1}{2}\right)^{2n+\frac{v}{k}+1} \times \left( I_{x,\infty}^{\zeta,\zeta',\delta,\delta',\tau} \left[ t^{\frac{\eta}{k}+\frac{v}{k}-2m\mu-\mu\nu+2n+1-1} \right] \right)(x)
\end{aligned} \tag{2.9}$$

On applying lemma (1.29) in Eq.(2.9),we get

$$\begin{aligned}
\Delta_2 = & x^{-\mu\nu+\frac{\eta}{k}+\frac{v}{k}+\tau-\zeta-\zeta'} \left(\frac{1}{2}\right)^{\nu+\frac{v}{k}+1} \frac{\sqrt{\pi}}{\Gamma(\nu+1)} \\
& \times \sum_{m=0}^{\infty} \frac{B(m+\frac{1}{2}, \nu+\frac{1}{2}; p, q)}{m!(\frac{1}{2})_m \Gamma(m+\frac{1}{2}) B(\frac{1}{2}, \nu+\frac{1}{2})} \left(\frac{-1}{4x^{2\mu}}\right)^m \\
& \times \sum_{n=0}^{\infty} \frac{1}{\Gamma_k\left(nk+v+\frac{3k}{2}\right)\Gamma\left(n+\frac{3}{2}\right)} \left(\frac{-cx^2}{4}\right)^n \frac{\Gamma\left(-\frac{\eta}{k}-\frac{v}{k}+2m\mu+\mu\nu-2n-\delta\right)}{\Gamma\left(-\frac{\eta}{k}-\frac{v}{k}+2m\mu+\mu\nu-2n\right)} \\
& \times \frac{\Gamma\left(-\frac{\eta}{k}-\frac{v}{k}+2m\mu+\mu\nu-2n-\tau+\zeta+\zeta'\right)}{\Gamma\left(-\frac{\eta}{k}-\frac{v}{k}+2m\mu+\mu\nu-2n-\tau+\zeta+\zeta'+\delta'\right)} \\
& \times \frac{\Gamma\left(-\frac{\eta}{k}-\frac{v}{k}+2m\mu+\mu\nu-2n-\tau+\zeta+\delta'\right)}{\Gamma\left(-\frac{\eta}{k}-\frac{v}{k}+2m\mu+\mu\nu-2n+\zeta-\delta\right)}
\end{aligned} \tag{2.10}$$

Now using Eq.(1.19) in (2.10) ,we get

$$\begin{aligned}
\Delta_2 = & x^{-\mu\nu+\frac{\eta}{k}+\frac{v}{k}+\tau-\zeta-\zeta'} k^{\tau+\frac{1}{2}} \left(\frac{1}{2}\right)^{\nu+\frac{v}{k}+1} \frac{\sqrt{\pi}}{\Gamma(\nu+1)} \\
& \times \frac{1}{\sqrt{\pi}} \sum_{m=0}^{\infty} \frac{B(m+\frac{1}{2}, \nu+\frac{1}{2}; p, q)}{m!(\frac{1}{2})_m B(\frac{1}{2}, \nu+\frac{1}{2})} \left(\frac{1}{4x^{2\mu}}\right)^m \\
& \times \sum_{n=0}^{\infty} \frac{1}{\Gamma_k\left(nk+v+\frac{3k}{2}\right)\Gamma_k\left(nk+\frac{3k}{2}\right)} \left(\frac{-cx^2k}{4}\right)^n
\end{aligned}$$

$$\times\frac{\Gamma_k\left(-\eta-v+2m\mu k+\mu\nu k-2nk-\delta k\right)}{\Gamma_k\left(-\eta-v+2m\mu k+\mu\nu k-2nk\right)}$$
$$\times\frac{\Gamma_k\left(-\eta-v+2m\mu k+\mu\nu k-2nk-\tau k+\zeta k+\zeta' k\right)}{\Gamma_k\left(-\eta-v+2m\mu k+\mu\nu k-2nk-\tau k+\zeta k+\zeta' k+\delta' k\right)}$$
$$\times\frac{\Gamma_k\left(-\eta-v+2m\mu k+\mu\nu k-2nk-\tau k+\zeta k+\delta' k\right)}{\Gamma_k\left(-\eta-v+2m\mu k+\mu\nu k-2nk+\zeta k-\delta k\right)} \tag{2.11}$$

Now using Eq.(1.24) in Eq.(2.11), then we get

$$\begin{aligned}\Delta_2 =& x^{-\mu\nu+\frac{\eta}{k}+\frac{v}{k}+\tau-\zeta-\zeta'}k^{\tau+\frac{1}{2}}\left(\frac{1}{2}\right)^{\nu+\frac{v}{k}+1}\frac{1}{\Gamma(\nu+1)}\times {}_1F_2\left[\begin{matrix}\frac{1}{2}\\ \frac{1}{2},\nu+1\end{matrix}\middle|-\frac{1}{4x^{2\mu}};p,q\right]\\
&\times\sum_{n=0}^{\infty}\frac{\Gamma\left(n+1\right)}{\Gamma_k\left(nk+v+\frac{3k}{2}\right)\Gamma_k\left(nk+\frac{3k}{2}\right)n!}\left(\frac{-cx^2k}{4}\right)^n\\
&\times\frac{\Gamma_k\left(-\eta-v+2m\mu k+\mu\nu k-2nk-\delta k\right)}{\Gamma_k\left(-\eta-v+2m\mu k+\mu\nu k-2nk\right)}\\
&\times\frac{\Gamma_k\left(-\eta-v+2m\mu k+\mu\nu k-2nk-\tau k+\zeta k+\zeta' k\right)}{\Gamma_k\left(-\eta-v+2m\mu k+\mu\nu k-2nk-\tau k+\zeta k+\zeta' k+\delta' k\right)}\\
&\times\frac{\Gamma_k\left(-\eta-v+2m\mu k+\mu\nu k-2nk-\tau k+\zeta k+\delta' k\right)}{\Gamma_k\left(-\eta-v+2m\mu k+\mu\nu k-2nk+\zeta k-\delta k\right)}\end{aligned} \tag{2.12}$$

Using the definition of (1.26) in (2.12),we at once arrive at the desired result.

**Theorem 3.** *Let* $\zeta,\zeta',\delta,\delta',\tau,\frac{\eta}{k},\mu\in\mathbb{C}$ *and* $k>0$ *be such that* $\Re(\tau)>0,\Re(\mu)>0,\Re(\frac{\eta}{k})>max\{0,\Re\left(-\zeta+\delta\right),\Re\left(-\zeta-\zeta'-\delta'+\tau\right)\}$*. Also let* $c\in\Re; v>-1$*,then for* $t>0$

$$\begin{aligned}\left(D_{0,x}^{\zeta,\zeta',\delta,\delta',\tau}\left[t^{\frac{\eta}{k}-1}S_{v,c}^{k}(t)J_{\nu,p,q}(t^{\mu})\right]\right)(x)=&\frac{x^{-\tau+\zeta+\zeta'+\frac{\eta}{k}+\frac{v}{k}+\mu\nu}}{\Gamma(\nu+1)}k^{-\tau+\frac{1}{2}}\left(\frac{1}{2}\right)^{\nu+\frac{v}{k}+1}\\
&{}_1F_2\left[\begin{matrix}\frac{1}{2}\\ \frac{1}{2},\nu+1\end{matrix}\middle|-\frac{x^{2\mu}}{4};p,q\right]\times\end{aligned}$$

$$ {}_4\psi_5^k\left[\begin{matrix}(v+\eta+k+2\mu mk+\mu\nu k,2k),\\ (v+\eta+k-\tau k+\zeta k+\zeta' k+\delta' k+2\mu mk+\mu\nu k,2k),\\ (v+\eta+k+2\mu mk+\mu\nu k-\delta k+\zeta k,2k),(1,1) & ; & \\ (v+\eta+k+2\mu mk+\mu\nu k-\delta k,2k) & & -\dfrac{x^2ck}{4}\\ ,(v+\eta+k-\tau k+\zeta k+\zeta' k+2\mu mk+\mu\nu k,2k), \\ (v+\eta+k+2\mu mk+\mu\nu k-\tau k+\zeta k+\delta' k,2k),\\ (v+\frac{3k}{2},k),(\frac{3k}{2},k) & ; \end{matrix}\right] \tag{2.13}$$

**Proof.** We denotes the left-hand sides of theorem (3) by $\Delta_3$,

$$\Delta_3 = \left(D_{0,x}^{\zeta,\zeta',\delta,\delta',\tau}\left[t^{\frac{\eta}{k}-1}S_{v,c}^{k}(t)J_{\nu,p,q}(t^{\mu})\right]\right)(x) \tag{2.14}$$

Using formula (1.16) and (1.21) in the right -hand side of Eq.(2.14) then change the order of the Integration and summation,we find that

$$\begin{aligned}\Delta_3 =&\frac{\sqrt{\pi}}{\Gamma(\nu+1)}\sum_{m=0}^{\infty}\frac{(-1)^m B(m+\frac{1}{2},\nu+\frac{1}{2};p,q)}{m!\Gamma(m+\frac{1}{2})B(\frac{1}{2},\nu+\frac{1}{2})}\left(\frac{1}{2}\right)^{2m+\nu}\\ &\times\sum_{n=0}^{\infty}\frac{(-c)^n}{\Gamma_k\left(nk+v+\frac{3k}{2}\right)\Gamma\left(n+\frac{3}{2}\right)}\\ &\times\left(\frac{1}{2}\right)^{2n+\frac{v}{k}+1}\times\left(D_{0,x}^{\zeta,\zeta',\delta,\delta',\tau}\left[t^{\frac{\eta}{k}+\frac{v}{k}+2m\mu+\mu\nu+2n+1-1}\right]\right)(x)\end{aligned} \tag{2.15}$$

On applying lemma (1.30) in Eq.(2.15),we get

$$\begin{aligned}\Delta_3 =&x^{\mu\nu+\frac{\eta}{k}+\frac{v}{k}-\tau+\zeta+\zeta'}\left(\frac{1}{2}\right)^{\nu+\frac{v}{k}+1}\frac{\sqrt{\pi}}{\Gamma(\nu+1)}\sum_{m=0}^{\infty}\frac{B(m+\frac{1}{2},\nu+\frac{1}{2};p,q)}{m!\Gamma(m+\frac{1}{2})B(\frac{1}{2},\nu+\frac{1}{2})}\left(\frac{-x^{2\mu}}{4}\right)^{m}\\ &\times\sum_{n=0}^{\infty}\frac{1}{\Gamma_k\left(nk+v+\frac{3k}{2}\right)\Gamma\left(n+\frac{3}{2}\right)}\left(\frac{-cx^2}{4}\right)^n\\ &\times\frac{\Gamma\left(\frac{\eta}{k}+\frac{v}{k}+2m\mu+\mu\nu+2n+1\right)}{\Gamma\left(\frac{\eta}{k}+\frac{v}{k}+2m\mu+\mu\nu+2n+1-\delta\right)}\\ &\times\frac{\Gamma\left(\frac{\eta}{k}+\frac{v}{k}+2m\mu+\mu\nu+2n+1-\tau+\zeta+\zeta'+\delta'\right)}{\Gamma\left(\frac{\eta}{k}+\frac{v}{k}+2m\mu+\mu\nu+2n+1-\tau+\zeta+\zeta'\right)}\\ &\times\frac{\Gamma\left(\frac{\eta}{k}+\frac{v}{k}+2m\mu+\mu\nu+2n+1-\delta+\zeta\right)}{\Gamma\left(\frac{\eta}{k}+\frac{v}{k}+2m\mu+\mu\nu+2n+1-\tau+\zeta+\delta'\right)}\end{aligned} \tag{2.16}$$

Now using Eq.(1.19) in (2.16), we get

$$\begin{aligned}\Delta_3 =& x^{\mu\nu+\frac{\eta}{k}+\frac{v}{k}-\tau+\zeta+\zeta'}k^{-\tau+\frac{1}{2}}\left(\frac{1}{2}\right)^{\nu+\frac{v}{k}+1}\frac{\sqrt{\pi}}{\Gamma(\nu+1)}\times\frac{1}{\sqrt{\pi}}\\ &\times\sum_{m=0}^{\infty}\frac{B(m+\frac{1}{2},\nu+\frac{1}{2};p,q)}{m!(\frac{1}{2})_m B(\frac{1}{2},\nu+\frac{1}{2})}\left(\frac{-x^{2\mu}}{4}\right)^m\\ &\times\sum_{n=0}^{\infty}\frac{1}{\Gamma_k\left(nk+v+\frac{3k}{2}\right)\Gamma_k\left(nk+\frac{3k}{2}\right)}\left(\frac{-cx^2k}{4}\right)^n\end{aligned}$$

$$\times \frac{\Gamma_k\left(\eta+v+2m\mu k+\mu\nu k+2nk+k\right)}{\Gamma_k\left(\eta+v+2m\mu k+\mu\nu k+2nk+k-\delta k\right)}$$
$$\times \frac{\Gamma_k\left(\eta+v+2m\mu k+\mu\nu k+2nk+k-\tau k+\zeta k+\zeta' k+\delta' k\right)}{\Gamma_k\left(\eta+v+2m\mu k+\mu\nu k+2nk+k-\tau k+\zeta k+\zeta' k\right)}$$
$$\times \frac{\Gamma_k\left(\eta+v+2m\mu k+\mu\nu k+2nk+k-\delta k+\zeta k\right)}{\Gamma_k\left(\eta+v+2m\mu k+\mu\nu k+2nk+k-\tau k+\zeta k+\delta' k\right)} \tag{2.17}$$

Now using Eq.(1.24) in Eq.(2.17) ,then we get

$$\begin{aligned}\Delta_3 =& x^{\mu\nu+\frac{\eta}{k}+\frac{v}{k}-\tau+\zeta+\zeta'}k^{-\tau+\frac{1}{2}}\left(\frac{1}{2}\right)^{\nu+\frac{v}{k}+1}\frac{1}{\Gamma(\nu+1)}\times {}_1F_2\left[\begin{array}{c}\frac{1}{2}\\ \frac{1}{2},\nu+1\end{array}\middle|\; -\frac{x^{2\mu}}{4};p,q\right]\\
&\times\sum_{n=0}^{\infty}\frac{\Gamma\left(n+1\right)}{\Gamma_k\left(nk+v+\frac{3k}{2}\right)\Gamma_k\left(nk+\frac{3k}{2}\right)n!}\left(\frac{-cx^2k}{4}\right)^n\\
&\times\frac{\Gamma_k\left(\eta+v+2m\mu k+\mu\nu k+2nk+k\right)}{\Gamma_k\left(\eta+v+2m\mu k+\mu\nu k+2nk+k-\delta k\right)}\\
&\times\frac{\Gamma_k\left(\eta+v+2m\mu k+\mu\nu k+2nk+k-\tau k+\zeta k+\zeta' k+\delta' k\right)}{\Gamma_k\left(\eta+v+2m\mu k+\mu\nu k+2nk+k-\tau k+\zeta k+\zeta' k\right)}\\
&\times\frac{\Gamma_k\left(\eta+v+2m\mu k+\mu\nu k+2nk+k-\delta k+\zeta k\right)}{\Gamma_k\left(\eta+v+2m\mu k+\mu\nu k+2nk+k-\tau k+\zeta k-\delta' k\right)}\end{aligned} \tag{2.18}$$

Using the definition of (1.26) in (2.18),we at once arrive at the desired result.

**Theorem 4.** *Let* $\zeta,\zeta',\delta,\delta',\tau,\frac{\eta}{k},\mu\in\mathbb{C}$ *and* $k>0, k\in\Re^+, |\frac{1}{t}|\leq 1$ *be such that* $\Re(\tau)>0,\Re(\mu)>0,\Re(\frac{\eta}{k})<1+min\{\Re(\delta'),\Re\left(\tau-\zeta-\zeta'\right),\Re\left(\tau-\zeta'-\delta\right)\}$. *Also let* $c\in\Re; v>-1$,*then for* $t>0$

$$\begin{aligned}\left(D^{\zeta,\zeta',\delta,\delta',\tau}_{x,\infty}\left[t^{\frac{\eta}{k}-1}S^k_{v,c}(t)J_{\nu,p,q}(\frac{1}{t^\mu})\right]\right)(x) =& \frac{x^{-\tau+\zeta+\zeta'+\frac{\eta}{k}+\frac{v}{k}-\mu\nu}}{\Gamma(\nu+1)}k^{-\tau+\frac{1}{2}}\left(\frac{1}{2}\right)^{\nu+\frac{v}{k}+1}\\
&{}_1F_2\left[\begin{array}{c}\frac{1}{2}\\ \frac{1}{2},\nu+1\end{array}\middle| -\frac{1}{4x^{2\mu}};p,q\right]\times\end{aligned}$$

$${}_4\psi_5^k\left[\begin{array}{cc}\begin{array}{c}(2\mu mk+\mu\nu k-\eta-v+\delta' k,-2k)\\ ,(2\mu mk+\mu\nu k-v-\eta+\tau k-\zeta k-\zeta' k,-2k),\\ (2\mu mk+\mu\nu k-\eta-v+\tau k-\zeta' k-\delta k,-2k),(1,1)\\ (2\mu mk+\mu\nu k-v-\eta,-2k),\\ (2\mu mk+\mu\nu k-v-\eta+\tau k-\zeta k-\zeta' k-\delta k,-2k),\\ (2\mu mk+\mu\nu k-v-\eta-\zeta' k+\delta' k,-2k),\\ (v+\frac{3k}{2},k),(\frac{3k}{2},k)\end{array} & \begin{array}{c}\\ \\ ;\\ \\ \\ \\ ;\end{array}\; -\frac{x^2ck}{4}\end{array}\right] \tag{2.19}$$

**Proof.** We denotes the left-hand sides of theorem (2) by $\Delta_4$,

$$\Delta_4 = \left( D_{x,\infty}^{\zeta,\zeta',\delta,\delta',\tau} \left[ t^{\frac{\eta}{k}-1} S_{v,c}^{k}(t) J_{\nu,p,q}(\frac{1}{t^{\mu}}) \right] \right)(x) \tag{2.20}$$

Using formula (1.16) and (1.21) in the right -hand side of Eq.(2.20) then change the order of the Integration and summation, we find that

$$\begin{aligned} \Delta_{4]} =& \frac{\sqrt{\pi}}{\Gamma(\nu+1)} \sum_{m=0}^{\infty} \frac{(-1)^m B(m+\frac{1}{2}, \nu+\frac{1}{2}; p, q)}{m! \Gamma(m+\frac{1}{2}) B(\frac{1}{2}, \nu+\frac{1}{2})} \left(\frac{1}{2}\right)^{2m+\nu} \\ &\times \sum_{n=0}^{\infty} \frac{(-c)^n}{\Gamma_k\left(nk+v+\frac{3k}{2}\right) \Gamma\left(n+\frac{3}{2}\right)} \\ &\times \left(\frac{1}{2}\right)^{2n+\frac{v}{k}+1} \times \left( D_{x,\infty}^{\zeta,\zeta',\delta,\delta',\tau} \left[ t^{\frac{\eta}{k}+\frac{v}{k}-2m\mu-\mu\nu+2n+1-1} \right] \right)(x) \end{aligned} \tag{2.21}$$

On applying lemma (1.31) in Eq.(2.21),we get

$$\begin{aligned} \Delta_4 =& x^{-\mu\nu+\frac{\eta}{k}+\frac{v}{k}-\tau+\zeta+\zeta'} \left(\frac{1}{2}\right)^{\nu+\frac{v}{k}+1} \frac{\sqrt{\pi}}{\Gamma(\nu+1)} \sum_{m=0}^{\infty} \frac{B(m+\frac{1}{2}, \nu+\frac{1}{2}; p, q)}{m! (\frac{1}{2})_m \Gamma(m+\frac{1}{2}) B(\frac{1}{2}, \nu+\frac{1}{2})} \left(\frac{-1}{4x^{2\mu}}\right)^m \\ &\times \sum_{n=0}^{\infty} \frac{1}{\Gamma_k\left(nk+v+\frac{3k}{2}\right) \Gamma\left(n+\frac{3}{2}\right)} \left(\frac{-cx^2}{4}\right)^n \times \frac{\Gamma\left(-\frac{\eta}{k}-\frac{v}{k}+2m\mu+\mu\nu-2n+\delta'\right)}{\Gamma\left(-\frac{\eta}{k}-\frac{v}{k}+2m\mu+\mu\nu-2n\right)} \\ &\times \frac{\Gamma\left(-\frac{\eta}{k}-\frac{v}{k}+2m\mu+\mu\nu-2n+\tau-\zeta-\zeta'\right)}{\Gamma\left(-\frac{\eta}{k}-\frac{v}{k}+2m\mu+\mu\nu-2n+\tau-\zeta-\zeta'-\delta\right)} \\ &\times \frac{\Gamma\left(-\frac{\eta}{k}-\frac{v}{k}+2m\mu+\mu\nu-2n+\tau-\zeta'-\delta\right)}{\Gamma\left(-\frac{\eta}{k}-\frac{v}{k}+2m\mu+\mu\nu-2n+\zeta'+\delta'\right)} \end{aligned} \tag{2.22}$$

Now using Eq.(1.19) in (2.22), we get

$$\begin{aligned} \Delta_4 =& x^{-\mu\nu+\frac{\eta}{k}+\frac{v}{k}-\tau+\zeta+\zeta'} k^{-\tau+\frac{1}{2}} \left(\frac{1}{2}\right)^{\nu+\frac{v}{k}+1} \frac{\sqrt{\pi}}{\Gamma(\nu+1)} \\ &\times \frac{1}{\sqrt{\pi}} \sum_{m=0}^{\infty} \frac{B(m+\frac{1}{2}, \nu+\frac{1}{2}; p, q)}{m! (\frac{1}{2})_m B(\frac{1}{2}, \nu+\frac{1}{2})} \left(\frac{-1}{4x^{2\mu}}\right)^m \\ &\times \sum_{n=0}^{\infty} \frac{1}{\Gamma_k(nk+v+\frac{3k}{2}) \Gamma_k(nk+\frac{3k}{2})} \left(\frac{-cx^2k}{4}\right)^n \end{aligned}$$

$$\times \frac{\Gamma_k\left(-\eta - v + 2m\mu k + \mu\nu k - 2nk + \delta' k\right)}{\Gamma_k\left(-\eta - v + 2m\mu k + \mu\nu k - 2nk\right)}$$
$$\times \frac{\Gamma_k\left(-\eta - v + 2m\mu k + \mu\nu k - 2nk + \tau k - \zeta k - \zeta' k\right)}{\Gamma_k\left(-\eta - v + 2m\mu k + \mu\nu k - 2nk + \tau k - \zeta k - \zeta' k - \delta k\right)} \tag{2.23}$$
$$\times \frac{\Gamma_k\left(-\eta - v + 2m\mu k + \mu\nu k - 2nk + \tau k - \zeta' k - \delta k\right)}{\Gamma_k\left(-\eta - v + 2m\mu k + \mu\nu k - 2nk - \zeta' k + \delta' k\right)}$$

Now using Eq.(1.24) in Eq.(2.23) ,then we get

$$\Delta_4 = x^{-\mu\nu + \frac{\eta}{k} + \frac{v}{k} - \tau + \zeta + \zeta'} k^{-\tau + \frac{1}{2}} \left(\frac{1}{2}\right)^{\nu + \frac{v}{k} + 1} \frac{1}{\Gamma(\nu+1)} {}_1F_2\left[\begin{array}{c} \frac{1}{2} \\ \frac{1}{2}, \nu + 1 \end{array} \middle| \frac{-1}{4x^{2\mu}}; p, q\right]$$
$$\times \sum_{n=0}^{\infty} \frac{1}{\Gamma_k(nk + v + \frac{3k}{2})\Gamma_k(nk + \frac{3k}{2})} \left(\frac{-cx^2k}{4}\right)^n$$
$$\times \frac{\Gamma_k\left(-\eta - v + 2m\mu k + \mu\nu k - 2nk + \delta' k\right)}{\Gamma_k\left(-\eta - v + 2m\mu k + \mu\nu k - 2nk\right)}$$
$$\times \frac{\Gamma_k(-\eta - v + 2m\mu k + \mu\nu k - 2nk + \tau k - \zeta k - \zeta' k)}{\Gamma_k(-\eta - \nu + 2m\mu k + \mu\nu k - 2nk + \tau k - \zeta k - \zeta' k - \delta k)}$$
$$\times \frac{\Gamma_k\left(-\eta - v + 2m\mu k + \mu\nu k - 2nk + \tau k - \zeta' k - \delta k\right)}{\Gamma_k\left(-\eta - v + 2m\mu k + \mu\nu k - 2nk - \zeta' k + \delta' k\right)} \tag{2.24}$$

Using the definition of (1.26) in (2.24),we at once arrive at the desired result.

## 3. Application

**Corollary 1.** *If we put $k = 1$ and $c = 1$ in Theorem 1, then it reduces to the following Struve function of order $v$, so we get the following results:*

$$\left(I_{0,x}^{\zeta,\zeta',\delta,\delta',\tau}\left[t^{\frac{\eta}{k}-1} H_v(t) J_{\nu,p,q}(t^\mu)\right]\right)(x)$$
$$= \frac{x^{\tau - \zeta - \zeta' + \eta + v + \mu\nu}}{\Gamma(\nu+1)} \left(\frac{1}{2}\right)^{\nu + v + 1} {}_1F_2\left[\begin{array}{c} \frac{1}{2} \\ \frac{1}{2}, \nu + 1 \end{array} \middle| -\frac{x^{2\mu}}{4}; p, q\right]$$
$${}_4\psi_5^k\left[\begin{array}{cc} \begin{array}{c}(v + \eta + 1 + 2\mu m + \mu\nu, 2), \\ (v + \eta + 1 + \tau - \zeta - \zeta' - \delta + 2\mu m + \mu\nu, 2), \\ (v + \eta + 1 + 2\mu m + \mu\nu + \delta' - \zeta', 2), (1, 1)\end{array} & ; \\ & -\dfrac{x^2c}{4} \\ \begin{array}{c}(v + \eta + 1 + 2\mu m + \mu\nu + \delta', 2), \\ (v + \eta + 1 + \tau - \zeta - \zeta' + 2\mu m + \mu\nu, 2), \\ (v + \eta + 1 + 2\mu m + \mu\nu + \tau - \zeta' - \delta, 2), \\ (v + \frac{3}{2}, 1), (\frac{3}{2}, 1)\end{array} & ; \end{array}\right] \tag{3.1}$$

**Corollary 2.** *If we put $k=1$ and $c=1$ in Theorem 2, then it reduces to the following Struve function of order $v$, so we get the following results:*

$$\left(I^{\zeta,\zeta',\delta,\delta',\tau}_{x,\infty}\left[t^{\frac{\eta}{k}-1}H_v(t)J_{\nu,p,q}(\frac{1}{t^{\mu}})\right]\right)(x)$$
$$=\frac{x^{\tau-\zeta-\zeta'+\eta+v-\mu\nu}}{\Gamma(\nu+1)}\left(\frac{1}{2}\right)^{\nu+v+1}{}_1F_2\left[\begin{array}{c}\frac{1}{2}\\ \frac{1}{2},\nu+1\end{array}\Bigg|-\frac{1}{4x^{2\mu}};p,q\right]$$
$${}_4\psi_5\left[\begin{array}{cc}\begin{array}{c}(2\mu m+\mu\nu-\eta-v-\delta,-2),(2\mu m+\mu\nu-v-\eta-\tau+\zeta+\zeta',-2),\\(2\mu m+\mu\nu-\eta-v-\tau+\zeta+\delta',-2),(1,1)\end{array} & ; \\ \begin{array}{c}(2\mu m+\mu\nu-v-\eta,-2),(2\mu m+\mu\nu-v-\eta-\tau+\zeta+\zeta'+\delta',-2),\\(2\mu m+\mu\nu-v-\eta+\zeta-\delta,-2),(v+\frac{3}{2},1),(\frac{3}{2},1)\end{array} & ;\end{array}-\frac{x^2c}{4}\right] \tag{3.2}$$

**Corollary 3.** *If we put $k=1$ and $c=1$ in Theorem 3, then it reduces to the following Struve function of order $v$,s o we get the following results:*

$$\left(D^{\zeta,\zeta',\delta,\delta',\tau}_{0,x}\left[t^{\frac{\eta}{k}-1}H_v(t)J_{\nu,p,q}(t^{\mu})\right]\right)(x)$$
$$=\frac{x^{-\tau+\zeta+\zeta'+\eta+v+\mu\nu}}{\Gamma(\nu+1)}\left(\frac{1}{2}\right)^{\nu+v+1}{}_1F_2\left[\begin{array}{c}\frac{1}{2}\\ \frac{1}{2},\nu+1\end{array}\Bigg|-\frac{x^{2\mu}}{4};p,q\right]$$
$${}_4\psi_5\left[\begin{array}{cc}\begin{array}{c}(v+\eta+1+2\mu m+\mu\nu,2),(v+\eta+1-\tau+\zeta+\zeta'+\delta'+2\mu m+\mu\nu,2),\\(v+\eta+1+2\mu m+\mu\nu-\delta+\zeta,2),(1,1)\end{array} & ; \\ \begin{array}{c}(v+\eta+1+2\mu m+\mu\nu-\delta,2),(v+\eta+1-\tau+\zeta+\zeta'+2\mu m+\mu\nu,2),\\(v+\eta+1+2\mu m+\mu\nu-\tau+\zeta+\delta',2),(v+\frac{3}{2},1),(\frac{3}{2},1)\end{array} & ;\end{array}-\frac{x^2c}{4}\right] \tag{3.3}$$

**Corollary 4.** *If we put $k=1$ and $c=1$ in Theorem 4, then it reduces to the following Struve function of order $v$, so we get the following results:*

$$\left(D^{\zeta,\zeta',\delta,\delta',\tau}_{x,\infty}\left[t^{\frac{\eta}{k}-1}H_v(t)J_{\nu,p,q}(\frac{1}{t^{\mu}})\right]\right)(x)$$
$$=\frac{x^{-\tau+\zeta+\zeta'+\eta+v-\mu\nu}}{\Gamma(\nu+1)}\left(\frac{1}{2}\right)^{\nu+v+1}{}_1F_2\left[\begin{array}{c}\frac{1}{2}\\ \frac{1}{2},\nu+1\end{array}\Bigg|-\frac{1}{4x^{2\mu}};p,q\right]$$
$${}_4\psi_5\left[\begin{array}{cc}\begin{array}{c}(-v-\eta+2\mu m+\mu\nu+\delta',-2),(-v-\eta+\tau-\zeta-\zeta'+2\mu m+\mu\nu,-2),\\(-v-\eta+\tau+2\mu m+\mu\nu-\delta+\zeta',-2),(1,1)\end{array} & ; \\ \begin{array}{c}(-v-\eta+2\mu m+\mu\nu,-2),(-v-\eta+\tau-\zeta-\zeta'-\delta+2\mu m+\mu\nu,-2),\\(-v-\eta+2\mu m+\mu\nu-\zeta'+\delta',-2),(v+\frac{3}{2},1),(\frac{3}{2},1)\end{array} & ;\end{array}-\frac{x^2c}{4}\right] \tag{3.4}$$

**Other known results are below:**

- If we consider $J_{\nu,p,q}(t^{\mu}) = 1$ and $\eta = \lambda$ , Theorem 1 reduces to the known result due to Seema Kabra et.al([19],pp.596,Eq.(2.1)).
- If we consider $J_{\nu,p,q}(t^{\mu}) = 1$ and $\eta = \lambda$, Theorem 2 reduces to the known result due to Seema Kabra et.al([19],pp.597,Eq.(2.2)).
- If we consider $J_{\nu,p,q}(t^{\mu}) = 1$ and $\eta = \lambda$ ,Theorem 3 reduces to the known result due to by Seema Kabra et.al([19],pp.599,Eq.(2.5)).
- If we consider $J_{\nu,p,q}(t^{\mu}) = 1$ and $\eta = \lambda$ , Theorem 4 reduces to the known result due to Seema Kabra et.al([19],pp.599,Eq.(2.5)).
- If we consider $S^{k}_{v,c}(x) = 1$ and $\frac{\eta}{k} \to \sigma$, Theorem 1 reduces to the known result due to H. M. Srivastava et.al([45],pp.6,Eq.(28))
- If we consider $S^{k}_{v,c}(x) = 1$ and $\frac{\eta}{k} \to \sigma$, Theorem 2 reduces to the known result due to H. M. Srivastava et.al([45],pp.7,Eq.(31)).
- If we consider $S^{k}_{v,c}(x) = 1$ and $\frac{\eta}{k} \to \sigma$, theorem 3 reduces to the known result due to H. M. Srivastava et.al([45],pp.9,Eq.(40)).
- If we consider $S^{k}_{v,c}(x) = 1$ and $\frac{\eta}{k} \to \sigma$, Theorem 4 reduces to the known result due to by H. M. Srivastava et.al([45],pp.10,Eq.(43)).

## 4. Conclusion

In present paper, we find out the fractional calculus operator( Marichev- Saigo-Maeda fractional integral and differential operator) involving the product of the $(p, q)$-Extended Bessel function and Generalized k-Struve function. Now we are going to conclude of this paper by highlighting that our results. All the theorems and corollaries of section 2 and section 3 can be results for generalized k-Wright function and $(p, q)$ -extended Generalized hypergeometric function. We can simply obtain various known and new results in applications.

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