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CHARACTERIZATIONS OF δ_p-NORMAL SPACE BY GENERALIZED VERSION OF δ_p-OPEN SET IN FUZZY m-SPACE

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Abstract: In this paper we characterize fuzzy m-δ_p-normal space [7] by $fmg\delta_p$-open set, the class of which is strictly larger than that of fuzzy m-open set [2]. Also here we introduce fuzzy quasi (m, m_1)-δ-preclosed, almost $f(m, m_1)g\delta_p$-closed and $fm\delta_p$-$fm_1g\delta_p$-closed functions between two fuzzy m-spaces and establish the interrelations of these functions. The applications of these functions on fuzzy m-δ_p-normal space are shown here.

Keywords and Phrases: Fuzzy m-closed set, fuzzy m-δ-preclosed set, $fmg\delta_p$-closed set, fuzzy m-δ_p-normal space, fuzzy strongly (m, m_1)-δ-preclosed function, $fm\delta_p$-$fm_1g\delta_p$-closed function, fuzzy m-normal space.

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1. Introduction

Alimohammady and Roohi introduced fuzzy minimal structure as follows : A family $\mathcal{M}$ of fuzzy sets in a non-empty set X is said to be a fuzzy minimal structure on X if $\alpha 1_X \in \mathcal{M}$ for every $\alpha \in [0, 1]$ [1]. However a more general version of it is introduced in [8, 10] as follows : A family $\mathcal{F}$ of fuzzy sets in a non-empty set X is a fuzzy minimal structure on X if $0_X \in \mathcal{F}$ and $1_X \in \mathcal{F}$. In this paper, we use this notion of fuzzy minimal structure. In [2], we introduced fuzzy minimal space (fuzzy m-space, for short) as follows : Let X be a non-empty set and $m \subset I^X$.

Then (X, m) is called fuzzy m-space if $0_X \in m$ and $1_X \in m$. The members of m are called fuzzy m-open sets and the complement of a fuzzy m-open set is called fuzzy m-closed set [2]. In [3], fuzzy m-δ-preopen set is introduced. Using this concept as a basic tool, in [7] we introduce and study fuzzy m-δ_p-normal space. Here we further characterize fuzzy m-δ_p-normal space by introducing $fmg\delta_p$-open set, the class of which is strictly larger than that of fuzzy m-δ-preopen set.

2. Preliminaries

Throughout this paper, (X, m) or simply by X we shall mean a fuzzy minimal space (fuzzy m-space, for short). The support [12] of a fuzzy set A, denoted by $suppA$ or A_0 and is defined by $suppA = \{x \in X : A(x) \neq 0\}$. The fuzzy set with the singleton support $\{x\} \subseteq X$ and the value t $(0 < t \leq 1)$ will be denoted by x_t. 0_X and 1_X are the constant fuzzy sets taking values 0 and 1 respectively in X. The complement [12] of a fuzzy set A in a fuzzy m-space X is denoted by $1_X \setminus A$ and is defined by $(1_X \setminus A)(x) = 1 - A(x)$, for each $x \in X$. For any two fuzzy sets A, B in X, $A \leq B$ means $A(x) \leq B(x)$, for all $x \in X$ [12] while AqB means A is quasi-coincident (q-coincident, for short) [11] with B, i.e., there exists $x \in X$ such that $A(x) + B(x) > 1$. The negation of these two statements will be denoted by $A \not\leq B$ and $A \not{q} B$ respectively. For any two fuzzy sets A and B in a fuzzy m-space (X, m), $A \bigvee B = max\{A(x), B(x)\}$, for all $x \in X$ and $A \bigwedge B = min\{A(x), B(x)\}$, for all $x \in X$.

For a fuzzy set A and a fuzzy point x_α in X, $x_\alpha \in A$ means $A(x) \geq \alpha$.

3. Some Well-Known Definitions

In this section we recall some definitions from [2, 3, 4, 7] which will be used thereafter.

Definition 3.1. [2] *Let X be a non-empty set and $m \subset I^X$ an m-structure on X. For $A \in I^X$, the m-closure of A and m-interior of A are defined as follows :*

$$mclA = \bigwedge\{F : A \leq F, 1_X \setminus F \in m\}$$

$$mintA = \bigvee\{D : D \leq A, D \in m\}.$$

It can be observed that for a given fuzzy minimal structure on X, $A \in I^X$ does not imply that $mintA \in m$ and $mclA$ is fuzzy m-closed. But if m satisfies $\mathcal{B}$-condition (i.e., m is closed under arbitrary union) [3], then $mintA \in m$ and $mclA$ is fuzzy m-closed.

Lemma 3.2. [2] *Let (X, m) be a fuzzy m-space and $A \in I^X$. Then*
(i) $mclA = A$ if $1_X \setminus A \in m$ and
(ii) $mintA = A$ if $A \in m$.

Definition 3.3. [3] *Let (X, m) be a fuzzy m-space and $A \in I^X$. Then A is called fuzzy m-regular open (resp., fuzzy m-preopen) if $A = mint(mclA)$ (resp., $A \leq mint(mclA)$). The complement of this set is called fuzzy m-regular closed (resp., fuzzy m-preclosed).*
The intersection of all fuzzy m-preclosed sets containing a fuzzy set A is called fuzzy m-preclosure of A, denoted by $mpclA$.

Definition 3.4. [3] *Let (X, m) be a fuzzy m-space and $A \in I^X$. Then fuzzy m-δ-closure and fuzzy m-δ-interior of A, denoted by $m\delta clA$ and $m\delta intA$ are defined as follows :*

$$m\delta clA = \{x_\alpha \in X : Aqmint(mclU), \text{ for all } U \in m \text{ with } x_\alpha qU\},$$
$$m\delta intA = \bigvee\{W : W \text{ is fuzzy } m\text{-regular open set in } X \text{ with } W \leq A\}$$

Definition 3.5. [3] *Let (X, m) be a fuzzy m-space and $A \in I^X$. Then A is called fuzzy m-δ-preopen if $A \leq mint(m\delta clA)$.*
Then complement of this set is called fuzzy m-δ-preclosed.
The union (resp., intersection) of all fuzzy m-δ-preopen (resp., fuzzy m-δ-preclosed) sets contained in (resp., containing) a fuzzy set A in X is called fuzzy m-δ-preinterior (resp., fuzzy m-δ-preclosure) of A, denoted by $m\delta pintA$ (resp., $m\delta pclA$). In general, $m\delta pintA$ (resp., $m\delta pclA$) may not be fuzzy m-δ-preopen (resp., fuzzy m-δ-preclosed) for any $A \in I^X$. But if m satisfies $\mathcal{B}$ condition, then $m\delta intA$ (resp., $m\delta clA$) is fuzzy m-δ-preopen (resp., fuzzy m-δ-preclosed).
If $A \in I^X$ is fuzzy m-δ-preopen (resp., fuzzy m-δ-preclosed), then $m\delta pintA = A$ (resp., $m\delta pclA = A$). The collection of all fuzzy m-δ-preopen (resp., fuzzy m-δ-preclosed) sets in (X, m) will be denoted by $Fm\delta PO(X)$ (resp., $Fm\delta PC(X)$).

Result 3.6. [7] *let (X, m) be a fuzzy m-space and $A \in I^X$ and x_α be a fuzzy point in X. Then $x_\alpha \in m\delta pclA$ if and only if for any $U \in Fm\delta PO(X)$ with $x_\alpha qU$, UqA.*

Result 3.7. [7] *Let (X, m) be a fuzzy m-space and $A \in I^X$. Then for any $U \in Fm\delta PO(X)$, $U \not{q} A \Rightarrow U \not{q} m\delta pclA$.*

Lemma 3.8. [7] *Let (X, m) be a fuzzy m-space and $A, B \in I^X$. Then the following statements are true :*
(i) $m\delta pcl(1_X \setminus A) = 1_X \setminus m\delta pintA$,
(ii) $m\delta pint(1_X \setminus A) = 1_X \setminus m\delta pclA$,
(iii) if $A \leq B$, then (a) $m\delta pclA \leq m\delta pclB$, (b) $m\delta pintA \leq m\delta pintB$,
(iv) $m\delta pcl(m\delta pclA) = m\delta pclA$,
(v) $m\delta pint(m\delta pintA) = m\delta pintA$.

Definition 3.9. [3] *A function $f : (X, m) \to (Y, m_1)$ is said to be fuzzy (m, m_1)-open (resp., fuzzy (m, m_1)-closed) function if $f(U) \in m_1$ (resp., $f(U) \in m_1^c$) for* *every* $U \in m$ *(resp.,* $U \in m^c$*).*

Definition 3.10. [3] *A function* $f : (X, m) \to (Y, m_1)$ *is said to be fuzzy* (m, m_1)*-continuous if* $f^{-1}(U) \in m$ *for every* $U \in m_1$.

Definition 3.11. [7] *A function* $f : (X, m) \to (Y, m_1)$ *is said to be fuzzy strongly* (m, m_1)*-*δ*-preclosed function if* $f(U) \in Fm_1\delta PC(Y)$ *for every* $U \in Fm\delta PC(X)$.

Definition 3.12. [3] *A fuzzy* m*-space* (X, m) *is said to be fuzzy* m*-normal space if for any fuzzy* m*-closed set* F *and a fuzzy* m*-open set* G *such that* $F \leq G$*, there exists a fuzzy* m*-open set* H *such that* $F \leq H \leq mclH \leq G$.

Definition 3.13. [7] *A fuzzy* m*-space* (X, m) *is said to be fuzzy* m*-*δ_p*-normal space if for any fuzzy* m*-closed set* F *and a fuzzy* m*-open set* G *such that* $F \leq G$*, there exists a fuzzy* m*-*δ*-preopen set* H *such that* $F \leq H \leq m\delta pclH \leq G$.

4. $fmg\delta_p$-Closed Set : Some Properties

In this section we first introduce a generalized version of fuzzy closed set in fuzzy m-space and establish the mutual relationships of this newly defined set with the sets defined in [4, 6]. Next we introduce different types of functions and lastly we establish the mutual relationships among themselves.

Definition 4.1. *A fuzzy set* A *in a fuzzy* m*-space* (X, m) *is said to be* fmg*-closed* [4] *(resp.,* $fmgp$*-closed* [6]*) if* $mclA \leq U$ *(resp.,* $mpclA \leq U$*) whenever* $A \leq U$ *where* $U \in m$*. The complement of* fmg*-closed (resp.,* $fmgp$*-closed) set in* X *is called* fmg*-open (resp.,* $fmgp$*-open).*

Definition 4.2. *A fuzzy set* A *in a fuzzy* m*-space* (X, m) *is said to be* $fmg\delta_p$*-closed if* $m\delta pclA \leq U$ *whenever* $A \leq U$ *where* $U \in m$*. The complement of an* $fmg\delta_p$*-closed set is* $fmg\delta_p$*-open.*

Remark 4.3. *So we have the following relationship among the sets defined so far:*

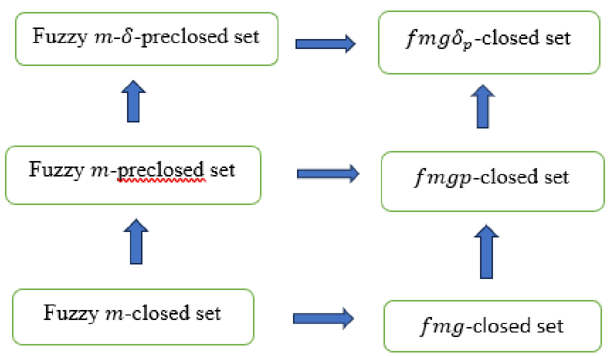

Figure 1

None of the above implications is reversible as it seen from the following examples.
Example 4.4. $fmg\delta_p$-closed set $\not\Rightarrow fmg$-closed set
Let $X = \{a, b\}$, $m = \{0_X, 1_X, A, B, C, D, E\}$ where $A(a) = 0.6, A(b) = 0.4$, $B(a) = B(b) = 0.5$, $C(a) = 0.5, C(b) = 0.4$, $D(a) = 0.6, D(b) = 0.5$, $E(a) = E(b) = 0.4$. Then (X, m) is a fuzzy m-space. Here $Fm\delta PO(X) = \{0_X, 1_X, P, Q, R, S\}$ where $P(a) = 0.4, P(b) \leq 0.4$, $0.4 < Q(a) \leq 0.6, Q(b) \leq 0.5$, $R \geq 1_X \setminus E$ and $0 \leq S(a) \leq 1, S(b) > 0.6$ and $Fm\delta PC(X)$= $\{0_X, 1_X, 1_X \setminus P, 1_X \setminus Q, 1_X \setminus R, 1_X \setminus S\}$ where $(1_X \setminus P)(a) = 0.6, (1_X \setminus P)(b) \geq 0.6$, $0.4 \leq (1_X \setminus Q)(a) < 0.6, (1_X \setminus Q)(b) \geq 0.5$, $1_X \setminus R \leq E$ and $0 \leq (1_X \setminus S)(a) \leq 1, (1_X \setminus S)(b) < 0.4$. Consider the fuzzy set U defined by $U(a) = 0.4, U(b) = 0.3$. Then $U < E$ where $E \in m$ and $mclU = 1_X \setminus D \not\leq E \Rightarrow U$ is not fmg-closed. But $m\delta pclU = U < E \Rightarrow U$ is $fmg\delta_p$-closed set in X.

Example 4.5. $fmg\delta_p$-closed set $\not\Rightarrow fmgp$-closed set
Consider Example 4.4. Here $FmPC(X)$= $\{0_X, 1_X, U, V\}$ where $U < E$ and $0.4 \leq V(a) \leq 0.5, V(b) = 0.5$. Consider the fuzzy set M defined by $M(a) = 0.51, M(b) = 0.5$. Then $M < D \in m$. Now $mpclM = 1_X \not\leq D \Rightarrow M$ is not $fmgp$-closed set in X. But $m\delta pclM = M < D \Rightarrow M$ is $fmg\delta_p$-closed set in X.

Example 4.6. $fmg\delta_p$-closed set $\not\Rightarrow$ fuzzy m-δ-preclosed set
Consider Example 4.4. Consider the fuzzy set W defined by $W(a) = 0.6, W(b) = 0.55$. Then W is not fuzzy m-δ-preclosed set in X. But $W < 1_X \in m$ only and so $m\delta pclW \leq 1_X \Rightarrow W$ is $fmg\delta_p$-closed set in X.

Definition 4.7. *A function $f : (X, m) \to (Y, m_1)$ is said to be*
(i) fuzzy (m, m_1)-δ-preclosed if $f(K)$ is fuzzy m_1-δ-preclosed set in Y for every fuzzy m-closed set K of X,
(ii) $f(m, m_1)g\delta_p$-closed if $f(K)$ is $fm_1g\delta_p$-closed in Y for every fuzzy m-closed set K of X.

Definition 4.8. *A function $f : (X, m) \to (Y, m_1)$ is said to be*
(i) fuzzy quasi (m, m_1)-δ-preclosed if $f(K) \in m_1^c$ for each $K \in Fm\delta PC(X)$,
(ii) $fm\delta_p$-$fm_1g\delta_p$-closed if $f(K)$ is $fm_1g\delta_p$-closed set in Y for each $K \in Fm\delta PC(X)$,
(iii) almost $f(m, m_1)g\delta_p$-closed if $f(K)$ is $fm_1g\delta_p$-closed set in Y for each $K \in FmRC(X)$.

Remark 4.9. *For a function $f : (X, m) \to (Y, m_1)$, the following diagram holds :*

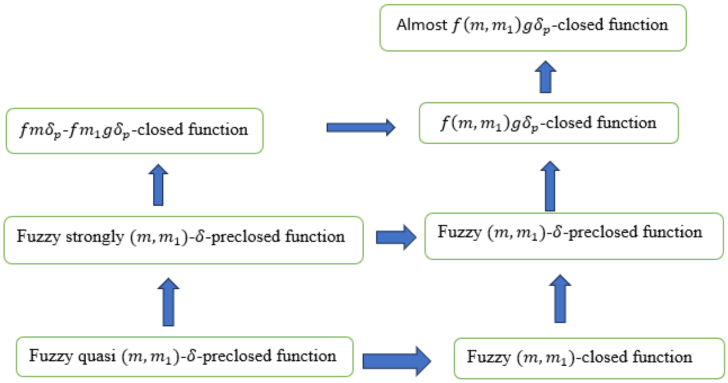

Figure 2

The following examples show that the reverse implications are not necessarily true, in general.

Example 4.10. Fuzzy (m, m_1)-closed function $\not\Rightarrow$ fuzzy quasi (m, m_1)-δ-preclosed function

Let $X = \{a, b\}$, $m = \{0_X, 1_X, A, B, C, D, E\}$, $m_1 = \{0_X, 1_X, A_1, B_1, C_1, D_1, E_1, F_1\}$ where $A(a) = 0.6, A(b) = 0.4,\ B(a) = B(b) = 0.5,\ C(a) = 0.5, C(b) = 0.4,$ $D(a) = 0.6, D(b) = 0.5,\ E(a) = E(b) = 0.4,\ A_1(a) = 0.4, A_1(b) = 0.5,\ B_1(a) =$ $0.5, B_1(b) = 0.4,\ C_1(a) = C_1(b) = 0.4,\ D_1(a) = D_1(b) = 0.5,\ E_1(a) = 0.6, E_1(b) =$ $0.4,\ F_1(a) = 0.6, F_1(b) = 0.5$. Then (X, m) and (X, m_1) are fuzzy m-spaces. Consider the identity function $i : (X, m) \to (X, m_1)$. Clearly i is fuzzy (m, m_1)-closed function. Consider the fuzzy set U defined by $U(a) = 0.6, U(b) = 0.7$. Then $U \in Fm\delta PC(X)$. Now $i(U) = U \notin m_1^c \Rightarrow i$ is not fuzzy quasi (m, m_1)-δ-preclosed function.

Example 4.11. Fuzzy strongly (m, m_1)-δ-preclosed function $\not\Rightarrow$ fuzzy quasi (m, m_1)-δ-preclosed function

Consider Example 4.10. Here $Fm\delta PC(X)= \{0_X, 1_X, U, V, W, S, T\}$ where $U(a) \geq 0.6, U(b) \geq 0.6,\ 0.5 \leq V(a) < 0.6, V(b) \geq 0.5,\ 0.4 \leq W(a) < 0.5, W(b) \geq 0.5,$ $S < E,\ 0 \leq T(a) \leq 1, T(b) < 0.4$ and that of $Fm_1\delta PC(X)$ is $\{0_X, 1_X, U_1, V_1, W_1\}$ where $U_1 \geq D_1$, $V_1 \geq A_1$, $W_1 < C_1$. Then clearly the identity function $i : (X, m) \to (X, m_1)$ is fuzzy strongly (m, m_1)-δ-preclosed function but not fuzzy quasi (m, m_1)-δ-preclosed function.

Example 4.12. Fuzzy (m, m_1)-δ-preclosed function $\not\Rightarrow$ fuzzy (m, m_1)-closed function

Let $X = \{a, b\}$, $m = \{0_X, 1_X, U\}$ where $U(a) = 0.4, U(b) = 0.3$, $m_1 = \{0_X, 1_X, A,$ $B, C, D, E\}$ where $A(a) = 0.6, A(b) = 0.4,\ B(a) = B(b) = 0.5,\ C(a) = 0.5, C(b) =$

0.4, $D(a) = 0.6, D(b) = 0.5$, $E(a) = E(b) = 0.4$. Then (X, m) and (X, m_1) are fuzzy m-spaces. Consider the identity function $i : (X, m) \to (X, m_1)$. Clearly i is not fuzzy (m, m_1)-closed function. But $1_X \setminus U \in m^c$, $i(1_X \setminus U) = 1_X \setminus U \in Fm_1\delta PC(X) \Rightarrow i$ is fuzzy (m, m_1)-δ-preclosed function.

Example 4.13. Fuzzy (m, m_1)-δ-preclosed function $\not\Rightarrow$ fuzzy strongly (m, m_1)-δ-preclosed function
Let $X = \{a, b\}$, $m = \{0_X, 1_X\}$, $m_1 = \{0_X, 1_X, A, B, C, D, E\}$ where $A(a) = 0.6, A(b) = 0.4, B(a) = B(b) = 0.5, C(a) = 0.5, C(b) = 0.4, D(a) = 0.6, D(b) = 0.5, E(a) = E(b) = 0.4$. Then (X, m) and (X, m_1) are fuzzy m-spaces. Consider the identity function $i : (X, m) \to (X, m_1)$. Clearly i is fuzzy (m, m_1)-δ-preclosed function. Now every fuzzy set in (X, m) is fuzzy m-δ-preclosed set in (X, m). Consider the fuzzy set S defined by $S(a) = 0.3, S(b) = 0.5$. Then $S \in Fm\delta PC(X)$. Now $i(S) = S \notin Fm_1\delta PC(X) \Rightarrow i$ is not fuzzy strongly (m, m_1)-δ-preclosed function.

Example 4.14. $f(m, m_1)g\delta_p$-closed function $\not\Rightarrow$ fuzzy (m, m_1)-δ-preclosed function
Let $X = \{a, b\}$, $m = \{0_X, 1_X, A, B, C, D, E\}$, $m_1 = \{0_X, 1_X, S\}$ where $A(a) = 0.6, (b) = 0.4, B(a) = B(b) = 0.5, C(a) = 0.5, C(b) = 0.4, D(a) = 0.6, D(b) = 0.5, E(a) = E(b) = 0.4, S(a) = 0.4, S(b) = 0.3$. Then (X, m) and (X, m_1) are fuzzy m-spaces. Consider the identity function $i : (X, m) \to (X, m_1)$. Here $1_X \setminus A \in m^c$, $i(1_X \setminus A) = 1_X \setminus A \notin Fm_1\delta PC(X)$ (Indeed, $Fm_1\delta PC(X) = \{0_X, 1_X, U, V\}$ where $U < S, V \geq 1_X \setminus S$) $\Rightarrow i$ is not fuzzy (m, m_1)-δ-preclosed function. But for any fuzzy m-closed set T in (X, m), we have $i(T) = T < 1_X \in m_1$ only and so $m_1\delta pclT \leq 1_X \Rightarrow i$ is $f(m, m_1)g\delta_p$-closed function.

Example 4.15. $f(m, m_1)g\delta_p$-closed function $\not\Rightarrow fm\delta_p$-$fm_1g\delta_p$-closed function
Let $X = \{a, b\}$, $m = \{0_X, 1_X, G\}$, $m_1 = \{0_X, 1_X, A, B, C, D, E\}$ where $G(a) = 0.5, G(b) = 0.4$, $A(a) = 0.6, A(b) = 0.4$, $B(a) = B(b) = 0.5$, $C(a) = 0.5, C(b) = 0.4$, $D(a) = 0.6, D(b) = 0.5$, $E(a) = E(b) = 0.4$. Then (X, m) and (X, m_1) are fuzzy m-spaces. Here $Fm\delta PC(X)= \{0_X, 1_X, S, T\}$ where $S \geq 1_X \setminus G$, $T < G$ and $Fm_1\delta PC(X) = \{0_X, 1_X, U, V, W, P\}$ where $U(a) = 0.6, U(b) \geq 0.6$, $0.5 \leq V(a) < 0.6, V(b) \geq 0.5$, $0.4 < W(a) < 0.5, W(b) \geq 0.5$, $P < E$. Consider the identity function $i : (X, m) \to (X, m_1)$. Let H be a fuzzy set in (X, m) defined by $H(a) = 0.5, H(b) = 0.3$. Then $H \in Fm\delta PC(X)$. Now $i(H) = H < C \in m_1$, but $m_1\delta pcl(H) = B \not\leq C \Rightarrow H$ is not $fm_1g\delta_p$-closed set in $(X, m_1) \Rightarrow i$ is not $fm\delta_p$-$fm_1g\delta_p$-closed function. But $1_X \setminus G \in m^c$, $i(1_X \setminus G) = 1_X \setminus G < 1_X(\in m_1)$ only and so $m_1\delta pcl(1_X \setminus G) \leq 1_X \Rightarrow 1_X \setminus G$ is $fm_1g\delta_p$-closed set in $(X, m_1) \Rightarrow i$ is $f(m, m_1)g\delta_p$-closed function.

Example 4.16. $fm_1\delta_p$-$fmg\delta_p$-closed function $\not\Rightarrow$ fuzzy strongly (m_1, m)-δ-preclosed function
Consider Example 4.15. Consider the identity function $i : (X, m_1) \to (X, m)$. For every fuzzy m_1-δ-preclosed set K other than $P(< E)$, 1_X is the only fuzzy m-open set in (X, m) such that $i(K) = K \leq 1_X$ and so $m\delta pclK \leq 1_X \Rightarrow K$ is $fmg\delta_p$-closed set in (X, m) and $i(P) = P < G \in m$ and so $m\delta pclP = P < G$. Hence i is $fm_1\delta_p$-$fmg\delta_p$-closed function. But $B \in Fm_1\delta PC(X)$, $i(B) = B \notin Fm\delta PC(X) \Rightarrow i$ is not fuzzy strongly (m_1, m)-δ-preclosed function.

Example 4.17. Almost $f(m, m_1)g\delta_p$-closed function $\not\Rightarrow f(m, m_1)g\delta_p$-closed function
Let $X = \{a, b\}$, $m = \{0_X, 1_X, G, H\}$, $m_1 = \{0_X, 1_X, A, B, C, D, E\}$ where $G(a) = 0.4, G(b) = 0.45$, $H(a) = 0.5, H(b) = 0.6$, $A(a) = 0.6, A(b) = 0.4$, $B(a) = B(b) = 0.5$, $C(a) = 0.5, C(b) = 0.4$, $D(a) = 0.6, D(b) = 0.5$, $E(a) = E(b) = 0.4$. Then (X, m) and (X, m_1) are fuzzy m-spaces. Now $FmRC(X) = \{0_X, 1_X, 1_X \setminus G\}$. Consider the identity function $i : (X, m) \to (X, m_1)$. Now $i(1_X \setminus G) = 1_X \setminus G < 1_X(\in m_1)$ only and so $m_1\delta pcl(1_X \setminus G) \leq 1_X \Rightarrow i$ is almost $f(m, m_1)g\delta_p$-closed function. But $1_X \setminus H \in m^c$ and $i(1_X \setminus H) = 1_X \setminus H \leq C \in m_1$ and $m_1\delta pcl(1_X \setminus H) = B \not\leq C \Rightarrow i$ is not $f(m, m_1)g\delta_p$-closed function.

Definition 4.18. *A function $f : (X, m) \to (Y, m_1)$ is said to be $fm\delta_p$-$fm_1g\delta_p$-continuous if $f^{-1}(K)$ is $fmg\delta_p$-closed set in X for every $K \in Fm_1\delta PC(Y)$.*

Theorem 4.19. *A function $f : (X, m) \to (Y, m_1)$ is $fm\delta_p$-$fm_1g\delta_p$-continuous if and only if $f^{-1}(K)$ is $fmg\delta_p$-open set in X for every $K \in Fm_1\delta PO(Y)$.*
Proof. Obvious.

Theorem 4.20. *If $f : (X, m) \to (Y, m_1)$ is fuzzy (m, m_1)-closed, $fm\delta_p$-$fm_1g\delta_p$-continuous, surjective function, then $f^{-1}(K)$ is $fmg\delta_p$-closed set in X for each $fm_1g\delta_p$-closed set K of Y, where m_1 satisfies $\mathcal{B}$ condition.*
Proof. Let K be $fm_1g\delta_p$-closed set in Y and let U be fuzzy m-open set in X such that $f^{-1}(K) \leq U$. We have to show that $m\delta pclf^{-1}(K) \leq U$. Put $V = 1_Y \setminus f(1_X \setminus U)$. As f is fuzzy (m, m_1)-closed function, V is fuzzy m_1-open set in Y. Now

$$f^{-1}(V) = f^{-1}(1_Y \setminus f(1_X \setminus U)) = 1_X \setminus f^{-1}(f(1_X \setminus U)) \leq 1_X \setminus (1_X \setminus U) = U \quad (i)$$

Now $f(1_X \setminus U) \leq 1_Y \setminus f(U) \Rightarrow V = 1_Y \setminus f(1_X \setminus U) \geq f(U) \Rightarrow f(U) \leq V \Rightarrow K = f(f^{-1}(K)) \leq f(U) \leq V$. As K is $fm_1g\delta_p$-closed in Y, $m_1\delta pclK \leq V \Rightarrow K \leq m_1\delta pclK \leq V \Rightarrow f^{-1}(K) \leq f^{-1}(m_1\delta pclK) \leq f^{-1}(V) \leq U$ (by (i)). As f is $fm\delta_p$-$fm_1g\delta_p$-continuous function, $f^{-1}(m_1\delta pclK)$ is $fmg\delta_p$-closed set in X (as m_1

satisfies $\mathcal{B}$ condition) and hence $m\delta pcl(f^{-1}(m_1\delta pclK)) \leq U$ (by definition). Therefore, $m\delta pcl(f^{-1}(K)) \leq m\delta pcl(f^{-1}(m_1\delta pclK)) \leq U \Rightarrow f^{-1}(K)$ is $fmg\delta_p$-closed set in X.

Theorem 4.21. *A surjective function $f : (X, m) \to (Y, m_1)$ is $fm\delta_p$-$fm_1g\delta_p$-closed if and only if for each $B \in I^Y$ and each $U \in Fm\delta PO(X)$ with $f^{-1}(B) \leq U$, there exists an $fm_1g\delta_p$-open set V of Y such that $B \leq V$ and $f^{-1}(V) \leq U$.*

Proof. Let f be $fm\delta_p$-$fm_1g\delta_p$-closed surjective function. Let $B \in I^Y$ and $U \in Fm\delta PO(X)$ with $f^{-1}(B) \leq U$. Let $V = 1_Y \setminus f(1_X \setminus U)$. Then $f^{-1}(V) = f^{-1}(1_Y \setminus f(1_X \setminus U)) = 1_X \setminus f^{-1}(f(1_X \setminus U)) \leq 1_X \setminus (1_X \setminus U) = U$ and so $f^{-1}(V) \leq U$. Now $1_X \setminus f^{-1}(B) \geq 1_X \setminus U \Rightarrow f(1_X \setminus U) \leq f(1_X \setminus f^{-1}(B)) = 1_Y \setminus f(f^{-1}(B)) = 1_Y \setminus B \Rightarrow B \leq 1_Y \setminus f(1_X \setminus U) = V$. As f is $fm\delta_p$-$fm_1g\delta_p$-closed function, $f(1_X \setminus U)$ is $fm_1g\delta_p$-closed set in Y as $1_X \setminus U \in Fm\delta PC(X)$ and so V is $fm_1g\delta_p$-open set in Y.

Conversely, let $K \in Fm\delta PC(X)$. Now $f^{-1}(1_Y \setminus f(K)) \leq 1_X \setminus K \in Fm\delta PO(X)$. By assumption, there exists an $fm_1g\delta_p$-open set V of Y such that $1_Y \setminus f(K) \leq V$ and $f^{-1}(V) \leq 1_X \setminus K \Rightarrow f(K) \geq 1_Y \setminus V$ and $K \leq 1_X \setminus f^{-1}(V) = f^{-1}(1_Y \setminus V) \Rightarrow f(K) \leq f(f^{-1}(1_Y \setminus V)) = 1_Y \setminus V \Rightarrow f(K) = 1_Y \setminus V$ which is $fm_1g\delta_p$-closed in Y. Hence f is $fm\delta_p$-$fm_1g\delta_p$-closed function.

Theorem 4.22. *If $f : (X, m) \to (Y, m_1)$ is fuzzy (m, m_1)-continuous and $fm\delta_p$-$fm_1g\delta_p$-closed function, then $f(H)$ is $fm_1g\delta_p$-closed set in Y for each $fmg\delta_p$-closed set H in X, where m satisfies $\mathcal{B}$ condition.*

Proof. Let H be $fmg\delta_p$-closed in X and V be fuzzy m_1-open set in Y such that $f(H) \leq V$. Now $H \leq f^{-1}(f(H)) \leq f^{-1}(V)$ which is fuzzy m-open in X. As H is $fmg\delta_p$-closed in X, $m\delta pclH \leq f^{-1}(V)$ and so $f(m\delta pclH) \leq f(f^{-1}(V)) \leq V$. Since f is $fm\delta_p$-$fm_1g\delta_p$-closed function and $m\delta pclH \in Fm\delta PC(X)$ (as m satisfies $\mathcal{B}$ condition), $f(m\delta pclH)$ is $fm_1g\delta_p$-closed set in Y. Then $m_1\delta pcl(f(m\delta pclH)) \leq V$ and so $f(H) \leq f(m\delta pclH) \leq m_1\delta pcl(f(m\delta pclH)) \leq V \Rightarrow m_1\delta pcl(f(H)) \leq m_1\delta pcl(f(m\delta pclH))$ (by using Lemma 3.8(iv))) $\leq V \Rightarrow f(H)$ is $fm_1g\delta_p$-closed set in Y.

Definition 4.23. [5] *A function $f : (X, m) \to (Y, m_1)$ is said to be fuzzy (m, m_1)-δ-precontinuous if $f^{-1}(V) \in Fm\delta PO(X)$ for every $V \in Fm_1\delta PO(Y)$.*

Remark 4.24. *Clearly a fuzzy (m, m_1)-δ-precontinuous function is $fm\delta_p$-$fm_1g\delta_p$-continuous, but the converse may not be true as it seen from the following example.*

Example 4.25. $fm\delta_p$-$fm_1g\delta_p$-continuous function $\not\Rightarrow$ fuzzy (m, m_1)-δ- precontinuous function

Let $X = \{a, b\}$, $m_1 = \{0_X, 1_X, S\}$, $m = \{0_X, 1_X, A, B, C, D, E\}$ where $S(a) =$

$0.4, S(b) = 0.45$, $A(a) = 0.6, A(b) = 0.4$, $B(a) = B(b) = 0.5$, $C(a) = 0.5, C(b) = 0.4$, $D(a) = 0.6, D(b) = 0.5$, $E(a) = E(b) = 0.4$. Then (X, m) and (X, m_1) are fuzzy m-spaces. Now $Fm\delta PC(X) = \{0_X, 1_X, U, V, W, P, Q\}$ where $U(a) \geq 0.6, U(b) \geq 0.6$, $0.5 \leq V(a) < 0.6, V(b) \geq 0.5$, $0.4 \leq W(a) < 0.5, W(b) \geq 0.5$, $P < E$, $0 \leq Q(a) \leq 1, Q(b) < 0.4$ and that of $Fm_1\delta PC(X) = \{0_X, 1_X, U_1, V_1, W_1\}$ where $U_1 < S$, $V_1 \geq 1_X \setminus S$, $W_1 \not\geq S$. Let us consider the identity function $i : (X, m_1) \to (X, m)$. Now $B \in Fm\delta PC(X)$. But $i^{-1}(B) = B \notin Fm_1\delta PC(X) \Rightarrow$ i is not fuzzy (m, m_1)-δ-precontinuous function. Now we show that i is $fm\delta_p$-$fm_1g\delta_p$-continuous function. Now any $M \in Fm\delta PC(X)$, $i^{-1}(M) = M$. Then either $M < S(\in m_1)$ or $M < 1_X(\in m_1)$. If $M < S$, then $M \in Fm_1\delta PC(X) \Rightarrow m_1\delta pclM = M < S$. But if $M < 1_X$ only, then $m_1\delta pclM < 1_X$. So i is $fm\delta_p$-$fm_1g\delta_p$-continuous function.

Result 4.26. *If $f : (X, m) \to (Y, m_1)$ is fuzzy (m, m_1)-open, bijective, fuzzy (m, m_1)-δ-precontinuous function, then $f^{-1}(K)$ is $fmg\delta_p$-closed set in X for each $fm_1g\delta_p$-closed set K in Y.*

Proof. Let K be $fm_1g\delta_p$-closed set in Y. Let U be fuzzy m-open set in X such that $f^{-1}(K) \leq U$. As f is fuzzy (m, m_1)-open and surjective function, $K = f(f^{-1}(K)) \leq f(U)$ where $f(U)$ is fuzzy m_1-open set in Y. By hypothesis, $K \leq m_1\delta pclK \leq f(U)$. Again f is fuzzy (m, m_1)-δ-precontinuous and injective function, $f^{-1}(K) \leq m\delta pcl(f^{-1}(K)) \leq m\delta pcl(f^{-1}(m_1\delta pclK)) = f^{-1}(m_1\delta pclK) \leq f^{-1}(f(U)) = U \Rightarrow m\delta pcl(f^{-1}(K)) \leq U$ whenever $f^{-1}(K) \leq U \in m \Rightarrow f^{-1}(K)$ is $fmg\delta_p$-closed set in X.

Theorem 4.27. *Let $f : (X, m) \to (Y, m_1)$ and $g : (Y, m_1) \to (Z, m_2)$ be two functions. Then*
(i) if f is $fm\delta_p$-$fm_1g\delta_p$-closed and g is fuzzy (m_1, m_2)-continuous, $fm_1\delta_p$-$fm_2g\delta_p$-closed functions, then $g \circ f$ is $fm\delta_p$-$fm_2g\delta_p$-closed function,
(ii) if f is fuzzy strongly (m, m_1)-δ-preclosed and g is $fm_1\delta_p$-$fm_2g\delta_p$-closed functions, then $g \circ f$ is $fm\delta_p$-$fm_2g\delta_p$-closed function,
(iii) if f is fuzzy quasi (m, m_1)-δ-preclosed and g is $f(m_1, m_2)g\delta_p$-closed functions, then $g \circ f$ is $fm\delta_p$-$fm_2g\delta_p$-closed function.

Proof. (i) Let $K \in Fm\delta PC(X)$. As f is $fm\delta_p$-$fm_1g\delta_p$-closed function, $f(K)$ is $fm_1g\delta_p$-closed set in Y. By Theorem 4.22, $g(f(K)) = (g \circ f)(K)$ is $fm_2g\delta_p$-closed set in $Z \Rightarrow g \circ f$ is $fm\delta_p$-$fm_2g\delta_p$-closed function.
(ii) Let $K \in Fm\delta PC(X)$. As f is fuzzy strongly (m, m_1)-δ-preclosed function, $f(K) \in Fm_1\delta PC(Y)$. g being $fm_1\delta_p$-$fm_2g\delta_p$-closed function, $g(f(K)) = (g \circ f)(K)$ is $fm_2g\delta_p$-closed set in $Z \Rightarrow g \circ f$ is $fm\delta_p$-$fm_2g\delta_p$-closed function.
(iii) Let $K \in Fm\delta PC(X)$. As f is fuzzy quasi (m, m_1)-δ-preclosed, $f(K)$ is fuzzy

m_1-closed set in Y. g being $f(m_1,m_2)g\delta_p$-closed function, $g(f(K)) = (g\circ f)(K)$ is $fm_2g\delta_p$-closed set in $Z \Rightarrow g\circ f$ is $fm\delta_p$-$fm_2g\delta_p$-closed function.

Theorem 4.28. *Let $f:(X,m)\to(Y,m_1)$ and $g:(Y,m_1)\to(Z,m_2)$ be two functions and let the composition $g\circ f:(X,m)\to(Z,m_2)$ be $fm\delta_p$-$fm_2g\delta_p$-closed function. Then if f is fuzzy (m,m_1)-δ-precontinuous surjective function, then g is $fm_1\delta_p$-$fm_2g\delta_p$-closed function.*
Proof. Let $K\in Fm_1\delta PC(Y)$. As f is fuzzy (m,m_1)-δ-precontinuous surjective function, $f^{-1}(K)\in Fm\delta PC(X)$ and $(g\circ f)(f^{-1}(K)) = g(K)$ is $fm_2g\delta_p$-closed set in $Z\Rightarrow g$ is $fm_1\delta_p$-$fm_2g\delta_p$-closed function.

Theorem 4.29. *$f:(X,m)\to(Y,m_1)$ and $g:(Y,m_1)\to(Z,m_2)$ be two functions and let the composition $g\circ f:(X,m)\to(Z,m_2)$ be $fm\delta_p$-$fm_2g\delta_p$-closed function. Then if g is fuzzy (m_1,m_2)-closed, $fm_1\delta_p$-$fm_2g\delta_p$-continuous bijective function, then f is $fm\delta_p$-$fm_1g\delta_p$-closed function.*
Proof. Let $K\in Fm\delta PC(X)$. Then $(g\circ f)(K)$ is $fm_2g\delta_p$-closed set in Z. Again g is fuzzy (m_1,m_2)-closed, $fm_1\delta_p$-$fm_2g\delta_p$-continuous surjective function, by Theorem 4.20, $g^{-1}((g\circ f)(K)) = f(K)$ (as g is injective) is $fm_1g\delta_p$-closed set in $Y\Rightarrow f$ is $fm\delta_p$-$fm_1g\delta_p$-closed function.

5. Fuzzy m-δ_p-Normal Space : More Characterizations

In this section we characterize fuzzy m-δ_p-normal space by $fmg\delta_p$-open set.

Lemma 5.1. *A fuzzy set A in a fuzzy m-space (X,m) is $fmg\delta_p$-open if and only if $F\le m\delta pintA$ whenever $F\le A$ and $F\in m^c$.*
Proof. First suppose that A is $fmg\delta_p$-open set in X. Let $F\in m^c$ be such that $F\le A$. then $1_X\setminus A\le 1_X\setminus F\in m$. By hypothesis, $m\delta pcl(1_X\setminus A) = 1_X\setminus m\delta pintA\le 1_X\setminus F\Rightarrow F\le m\delta pintA$.
Next suppose that $F\le m\delta pintA$ whenever $F\le A, F\in m^c$. We have to show that A is $fmg\delta_p$-open set in X. Suppose $F\le A$, $F\in m^c$. Now $1_X\setminus m\delta pintA\le 1_X\setminus F\Rightarrow m\delta pcl(1_X\setminus A)\le 1_X\setminus F\in m\Rightarrow 1_X\setminus A$ is $fmg\delta_p$-closed set in $X\Rightarrow A$ is $fmg\delta_p$-open set in X.

Theorem 5.2. *Let (X,m) be a fuzzy m-space where m satisfies $\mathcal{B}$ condition. Then the following statements are equivalent :*
(a) X is fuzzy m-δ_p-normal space,
(b) for any two fuzzy m-closed sets A,B with $A\,\not{q}B$, there exist two $fmg\delta_p$-open sets U and V in X such that $A\le U$, $B\le$ and $U\,\not{q}V$,
(c) for each fuzzy m-closed set A and each fuzzy m-open set B containing A, there exists an $fmg\delta_p$-open set U in X such that $A\le U\le m\delta pclU\le B$,
(d) for each fuzzy m-closed set A and each fmg-open set B containing A, there

exists $U \in Fm\delta PO(X)$ *such that* $A \leq U \leq m\delta pclU \leq mintB$,
(e) for each fuzzy m*-closed set* A *and each* fmg*-open set* B *containing* A*, there exists an* $fmg\delta_p$*-open set* G *such that* $A \leq G \leq m\delta pclG \leq mintB$,
(f) for each fmg*-closed set* A *and fuzzy* m*-open set* B *containing* A*, there exists* $U \in Fm\delta PO(X)$ *such that* $mclA \leq U \leq m\delta pclU \leq B$,
(g) for each fmg*-closed set* A *and each fuzzy* m*-open set* B *containing* A*, there exists an* $fmg\delta_p$*-open set* G *such that* $mclA \leq G \leq m\delta pclG \leq B$.

Proof. (a) $\Rightarrow$ (b) Obvious as every fuzzy m-δ-preopen set is $fmg\delta_p$-open.
(b) $\Rightarrow$ (a) Let A, B be two fuzzy m-closed sets with $A \not qB$. By (b), there exist $fmg\delta_p$-open sets U, V in X such that $A \leq U$, $B \leq V$ and $U \not qV$. Then $1_X \setminus U \leq 1_X \setminus A$, $1_X \setminus V \leq 1_X \setminus B$ where $1_X \setminus U$ and $1_X \setminus V$ are $fmg\delta_p$-closed sets in X and $1_X \setminus A \in m$, $1_X \setminus B \in m$. Then by definition of $fmg\delta_p$-closed set, $m\delta pcl(1_X \setminus U) \leq 1_X \setminus A$, $m\delta pcl(1_X \setminus V) \leq 1_X \setminus B \Rightarrow 1_X \setminus m\delta pintU \leq 1_X \setminus A$, $1_X \setminus m\delta pintV \leq 1_X \setminus B \Rightarrow A \leq m\delta pintU$, $B \leq m\delta pintV$ where $m\delta pintU$ and $m\delta pintV$ are fuzzy m-δ-preopen sets in X (as m satisfies $\mathcal{B}$ condition) with $(m\delta pintU) \not q(m\delta pintV)$ (as $U \not qV$).
(b) $\Rightarrow$ (c) Let A be fuzzy m-closed and B, fuzzy m-open sets in X containing A. Then $1_X \setminus B \leq 1_X \setminus A \Rightarrow A \not q(1_X \setminus B)$ where $1_X \setminus B$ is fuzzy m-closed set in X. By (b), there exist $fmg\delta_p$-open sets U, V in X such that $A \leq U$, $1_X \setminus B \leq V$ and $U \not qV$. Then $1_X \setminus V \leq B$ where $1_X \setminus V$ is $fmg\delta_p$-closed and B is fuzzy m-open sets in X. By definition, $m\delta pcl(1_X \setminus V) \leq B$. Now $U \not qV \Rightarrow U \leq 1_X \setminus V$. So $A \leq U \leq m\delta pclU \leq m\delta pcl(1_X \setminus V) \leq B \Rightarrow A \leq U \leq m\delta pclU \leq B$.
(c) $\Rightarrow$ (b) Let A, B be two fuzzy m-closed sets in X with $A \not qB$. Then $A \leq 1_X \setminus B$ where $1_X \setminus B$ is fuzzy m-open set in X. By (c), there exists an $fmg\delta_p$-open set U in X such that $A \leq U \leq m\delta pclU \leq 1_X \setminus B \Rightarrow A \leq U$, $B \leq 1_X \setminus m\delta pclU = m\delta pint(1_X \setminus U)$ where $m\delta pint(1_X \setminus U)$ being fuzzy m-δ-preopen in X (as m satisfies $\mathcal{B}$ condition) is $fmg\delta_p$-open set in X with $U \not q(1_X \setminus m\delta pclU) \Rightarrow U \not q m\delta pint(1_X \setminus U)$.
(c) $\Rightarrow$ (d) Let A be fuzzy m-closed and B be fmg-open sets in X containing A. Then $1_X \setminus B \leq 1_X \setminus A$ where $1_X \setminus A \in m$. As $1_X \setminus B$ is fmg-closed set in X, $mcl(1_X \setminus B) \leq 1_X \setminus A \Rightarrow 1_X \setminus mintB \leq 1_X \setminus A \Rightarrow A \leq mintB$ and $mintB \in m$ (as m satisfies $\mathcal{B}$ condition). By (c), there exists an $fmg\delta_p$-open set G such that $A \leq G \leq m\delta pclG \leq mintB$. Since G is $fmg\delta_p$-open set, by Lemma 5.1, $A \leq m\delta pintG$. Take $U = m\delta pintG$. Then $U \in Fm\delta PO(X)$ (as m satisfies $\mathcal{B}$ condition) such that $A \leq U \leq m\delta pclU = m\delta pcl(m\delta pintG) \leq m\delta pclG \leq mintB$.
(d) $\Rightarrow$ (e) The proof is immediate as every fuzzy m-δ-preopen set is $fmg\delta_p$-open.
(e) $\Rightarrow$ (f) Let A be fmg-closed set in X and B, a fuzzy m-open set in X containing A. Then $mclA \leq B$ where $mclA \in m^c$ (as m satisfies $\mathcal{B}$ condition). Since fuzzy m-open set is fmg-open, by (e) there exists an $fmg\delta_p$-open set G such that

$mclA \leq G \leq m\delta pclG \leq B$. Since G is $fmg\delta_p$-open set and $mclA \leq G$, by Lemma 5.1 $mclA \leq m\delta pintG$. Put $U = m\delta pintG$. Then $mclA \leq U \leq m\delta pclU = m\delta pcl(m\delta pintG) \leq m\delta pclG \leq B$.
(f) $\Rightarrow$ (g) The proof is immediate as every fuzzy m-δ-preopen set is $fmg\delta_p$-open set.
(g) $\Rightarrow$ (c) The proof is immediate as every fuzzy m-closed set is fmg-closed.

Theorem 5.3. *If $f : (X, m) \to (Y, m_1)$ is a fuzzy (m, m_1)-continuous, fuzzy quasi (m, m_1)-δ-preclosed bijective function and X is fuzzy m-δ_p-normal space, then Y is fuzzy m_1-normal space.*
Proof. Let A, B be two fuzzy m-closed sets in Y with $A \not{q} B$. Then $f^{-1}(A), f^{-1}(B)$ are fuzzy m-closed sets in X with $f^{-1}(A) \not{q} f^{-1}(B)$. As X is fuzzy m-δ_p-normal space, there exist $U, V \in Fm\delta PO(X)$ such that $f^{-1}(A) \leq U$, $f^{-1}(B) \leq V$ and $U \not{q} V$. Let $W_1 = 1_Y \setminus f(1_X \setminus U)$, $W_2 = 1_Y \setminus f(1_X \setminus V)$. As f is fuzzy quasi (m, m_1)-δ-preclosed, $f(1_X \setminus U), f(1_X \setminus V)$ are fuzzy m_1-closed sets in Y and so W_1, W_2 are fuzzy m_1-open sets in Y. As f is surjective, $A \leq f(U)$, $B \leq f(V)$. Again we know that $f(1_X \setminus U) \leq 1_Y \setminus f(U) \Rightarrow W_1 = 1_Y \setminus f(1_X \setminus U) \geq f(U)$ and also $W_2 \geq f(V)$. So $A \leq W_1, B \leq W_2$. we claim that $W_1 \not{q} W_2$. Indeed, if there is some $y \in Y$ such that $W_1(y) + W_2(y) > 1$. Then as f being injective $1 - (1_X \setminus U)(f^{-1}(y)) + 1 - (1_X \setminus V)(f^{-1}(y)) > 1 \Rightarrow 1 - 1 + U(f^{-1}(y)) + 1 - 1 + V(f^{-1}(y)) > 1 \Rightarrow U(f^{-1}(y)) + V(f^{-1}(y)) > 1 \Rightarrow UqV$, a contradiction. Hence Y is fuzzy m_1-normal space.

Theorem 5.4. *Let $f : (X, m) \to (Y, m_1)$ be a fuzzy (m, m_1)-closed, $fm\delta_p$-$fm_1g\delta_p$-continuous, injective function where m satisfies $\mathcal{B}$ condition. If Y is fuzzy m_1-δ_p-normal space, then X is fuzzy m-δ_p-normal space.*
Proof. Let A, B be two fuzzy m-closed sets in X with $A \not{q} B$. As f is fuzzy (m, m_1)-closed function, $f(A), f(B) \in m_1^c$ with $f(A) \not{q} f(B)$. Indeed, if $f(A)qf(B)$, then there exists $y \in Y$ such that $[f(A)](y) + [f(B)](y) > 1 \Rightarrow A(f^{-1}(y)) + B(f^{-1}(y)) > 1$ (as f is injective) $\Rightarrow AqB$, a contradiction. As Y is fuzzy m_1-δ_p-normal space, there exist $U, V \in Fm_1\delta PO(Y)$ such that $f(A) \leq U$, $f(B) \leq V$ and $U \not{q} V$. As f is $fm\delta_p$-$fm_1g\delta_p$-continuous function, $f^{-1}(U), f^{-1}(V)$ are $fmg\delta_p$-open sets in X with $f^{-1}(U) \not{q} f^{-1}(V)$ and $A \leq f^{-1}(U), B \leq f^{-1}(V)$. By Theorem 5.2 (b) $\Rightarrow$ (a), X is fuzzy m-δ_p-normal space.

Theorem 5.5. *If $f : (X, m) \to (Y, m_1)$ is a fuzzy (m, m_1)-closed, fuzzy (m, m_1)-δ-precontinuous, injective function and Y is fuzzy m_1-δ_p-normal space, then X is fuzzy m-δ_p-normal space.*
Proof. The proof follows from Remark 4.24 and Theorem 5.4.

Lemma 5.6. *A surjective function* $f : (X, m) \to (Y, m_1)$ *is almost* $f(m, m_1)g\delta_p$*-closed if and only if for each* $B \in I^Y$ *and each* $U \in FmRO(X)$ *with* $f^{-1}(B) \leq U$*, there exists an* $fm_1g\delta_p$*-open set* V *of* Y *such that* $B \leq V$ *and* $f^{-1}(V) \leq U$*.*

Proof. First suppose that f is almost $f(m, m_1)g\delta_p$-closed function. Let $B \in I^Y$ and $U \in FmRO(X)$ with $f^{-1}(B) \leq U$. As f is surjective, $B = f(f^{-1}(B)) \leq f(U)$. Let $V = 1_Y \setminus f(1_X \setminus U)$. As f is almost $f(m, m_1)g\delta_p$-closed function, V is $fm_1g\delta_p$-open set in Y. Now $V = 1_Y \setminus f(1_X \setminus U) \geq 1_Y \setminus (1_Y \setminus f(U)) = f(U)$ and so $B \leq f(U) \leq V$. Again $f^{-1}(V) = f^{-1}(1_Y \setminus f(1_X \setminus U)) = 1_X \setminus f^{-1}(f(1_X \setminus U)) \leq 1_X \setminus (1_X \setminus U) = U$.

Conversely, let $K \in FmRC(X)$. Now $f^{-1}(1_Y \setminus f(K)) = 1_X \setminus f^{-1}(f(K)) \leq 1_X \setminus K \in FmRO(X)$. By assumption, there exists an $fm_1g\delta_p$-open set V of Y such that $1_Y \setminus f(K) \leq V$ and $f^{-1}(V) \leq 1_X \setminus K$. Then $f(K) \geq 1_Y \setminus V$ and $K \leq 1_X \setminus f^{-1}(V) = f^{-1}(1_Y \setminus V) \Rightarrow f(K) \leq f(f^{-1}(1_Y \setminus V)) \leq 1_Y \setminus V \Rightarrow f(K) = 1_Y \setminus V \Rightarrow f(K)$ is $fm_1g\delta_p$-closed set in Y. Hence f is almost $f(m, m_1)g\delta_p$-closed function.

Lemma 5.7. *If* $f : (X, m) \to (Y, m_1)$ *is almost* $f(m, m_1)g\delta_p$*-closed surjective function where* m_1 *satisfies* $\mathcal{B}$ *condition, then for each fuzzy* m_1*-closed set* C *of* Y *and each* $U \in FmRO(X)$ *with* $f^{-1}(C) \leq U$*, there exists* $V \in Fm_1\delta PO(Y)$ *such that* $C \leq V$ *and* $f^{-1}(V) \leq U$*.*

Proof. Let C be fuzzy m_1-closed set in Y and $U \in FmRO(X)$ with $f^{-1}(C) \leq U$. By Lemma 5.6, there exists an $fm_1g\delta_p$-open set V of Y such that $C \leq V$ and $f^{-1}(V) \leq U$. Now $1_Y \setminus V \leq 1_Y \setminus C$ where $1_Y \setminus C$ is fuzzy m_1-open set and $1_Y \setminus V$ is $fm_1g\delta_p$-closed set in $Y \Rightarrow m_1\delta pcl(1_Y \setminus V) \leq 1_Y \setminus C \Rightarrow 1_Y \setminus m_1\delta pintV \leq 1_Y \setminus C \Rightarrow C \leq m_1\delta pintV = W$ (say). Then $W \in Fm_1\delta PO(Y)$ (as m_1 satisfies $\mathcal{B}$ condition) and $f^{-1}(W) = f^{-1}(m_1\delta pintV) \leq f^{-1}(V) \leq U$.

Theorem 5.8. *Let* $f : (X, m) \to (Y, m_1)$ *be a fuzzy* (m, m_1)*-continuous, almost* $f(m, m_1)g\delta_p$*-closed, bijective function where* m *satisfies* $\mathcal{B}$ *condition. If* X *is fuzzy* m*-normal space, then* Y *is fuzzy* m_1*-*δ_p*-normal space.*

Proof. Let A, B be fuzzy m_1-closed sets in Y with $A \not{q} B$. Since f is fuzzy (m, m_1)-continuous function, $f^{-1}(A), f^{-1}(B)$ are fuzzy m-closed sets in X with $f^{-1}(A) \not{q} f^{-1}(B)$. Since X is fuzzy m-normal space, there are fuzzy m-open sets U_1, U_2 in X such that $f^{-1}(A) \leq U_1$, $f^{-1}(B) \leq U_2$ and $U_1 \not{q} U_2$. Let $G_1 = mintmclU_1, G_2 = mintmclU_2$. Then as m satisfies $\mathcal{B}$ condition, $G_1, G_2 \in FmRO(X)$ and $f^{-1}(A) \leq U_1 \leq mintmclU_1 = G_1$ and $f^{-1}(B) \leq U_2 \leq mintmclU_2 = G_2$. By Lemma 5.7, there exist $V_1, V_2 \in Fm_1\delta PO(Y)$ such that $A \leq V_1$, $B \leq V_2$ and $f^{-1}(V_1) \leq G_1$, $f^{-1}(V_2) \leq G_2$. We claim that $G_1 \not{q} G_2$. Indeed, $U_1 \not{q} U_2 \Rightarrow U_1 \not{q} mclU_2$ (as U_1 is fuzzy m-open) $\Rightarrow U_1 \not{q} mintmclU_2 \Rightarrow mclU_1 \not{q} mintmclU_2$ (as

$mintmclU_2$ is fuzzy m-open as m satisfies $\mathcal{B}$ condition) $\Rightarrow mintmclU_1 \not{q} mintmclU_2 \Rightarrow G_1 \not{q} G_2$. As f is surjective, $V_1 \leq f(G_1)$, $V_2 \leq f(G_2)$ and $V_1 \not{q} V_2$. Infact, if there is some $y \in Y$ such that $V_1(y) + V_2(y) > 1$, then $[f(G_1)](y) + [f(G_2)](y) > 1 \Rightarrow G_1(f^{-1}(y)) + G_2(f^{-1}(y)) > 1$ (as f is bijective) $\Rightarrow G_1 q G_2$, a contradiction.

Corollary 5.9. *If $f : (X, m) \to (Y, m_1)$ is a fuzzy (m, m_1)-continuous, fuzzy (m, m_1)-δ-preclosed bijective function and X is fuzzy m-normal space, then Y is fuzzy m_1-δ_p-normal space.*

Proof. The proof follows from Theorem 5.8 using the fact that fuzzy (m, m_1)-δ-preclosed function is almost $f(m, m_1)g\delta_p$-closed function.

6. Conclusion

Here we introduce and study different types of generalized version of fuzzy closed sets and closed types functions in fuzzy m-space. Using these types of sets we characterize fuzzy m-δ_p-normal space. Also applications of these sets and functions are established here.

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